

Definition and representation of discrete signals in MATLAB

1. Discrete signals: definitions and characteristics

A discrete-time defined signal, $x[n]$, is a function whose independent variable n is an integer and is usually represented by a sequence of numbers (samples). An example of a discrete signal is represented in Fig. 1. It is important to note that a discrete signal is not defined between two consecutive samples, which is why it cannot be assumed that the signal $x[n]$ is zero for noninteger values of variable n .

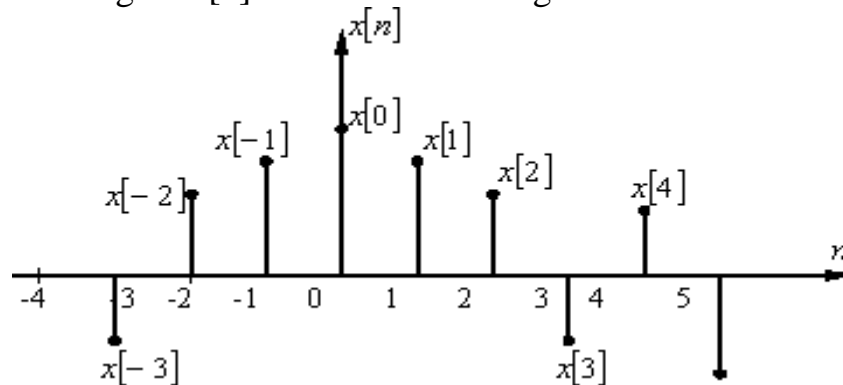


Figure 1. Graphical representation of a discrete signal $x[n]$

The mathematical model of a discrete signal can be defined as an application:

$$x: \mathbb{Z} \rightarrow \mathbb{C}, \quad n \rightarrow x[n]$$

A signal $x[n]$ is periodic, of period N , if and only if the following condition is met:

$$x[n \pm N] = x[n], \forall n \in \mathbb{Z} \text{ și } N \in \mathbb{N}$$

The smallest positive value of N , for which the preceding relation is fulfilled, is called *the fundamental period*. If there is no value for N to satisfy the relationship, the signal is called *nonperiodic* or *aperiodic*.

2. Definition of discrete signals in MATLAB

Discrete signals are defined in MATLAB as vectors or matrices, with each element in the vector or matrix representing the value of the signal being measured at a specific point in time or space.

If the signal is one-dimensional (represented by a single independent variable), a vector can be used to define it in MATLAB. Thus, each element of the vector will be associated with a certain discrete moment of time n and will reflect the amplitude of the signal at that specific moment.

If the signal is two-dimensional (represented by two independent variables), an array can be used to define it in MATLAB. In this case, each element of the matrix will correspond to a discrete signal value, and the size of the matrix will reflect the two-

dimensional structure of the signal. For example, to represent a digital image in MATLAB, one can create an array A containing the pixel values of the image:

A = [pixel_value1, pixel_value2, ...; pixel_valuen, ...].

Below, in Table 1, are presented some discrete signals, together with their definition in MATLAB.

Table 1. Definition of basic discrete signals in MATLAB

Discrete signal	Signal definition in MATLAB	Description
Unit Impulse (Dirac)	s=[1,zeros(1,N)]	generates a line vector that has the first element 1 and the next N elements are zeros
Unit step	s=ones(1,N)	generates a line vector with N elements equal to 1
Exponentially p^n	s=p.^n s=power(p,n)	generates an exponential sequence p^n , where $n = M:L, M, L \in \mathbb{Z}$
Sinusoidal	s=sin(2*pi*f*n)	generates a sine sequence with discrete frequency f, where $n = M:L, M, L \in \mathbb{Z}$
Rectangular	s=square(2*pi*f*n)	generates a sequence for a rectangular signal with discrete frequency f, where $n = M:L, M, L \in \mathbb{Z}$
Sawtooth	s=sawtooth(2*pi*f*n)	generates a sequence for a sawtooth signal with discrete frequency f, where $n = M:L, M, L \in \mathbb{Z}$

For the sinusoidal, rectangular and sawtooth signals, the use of *discrete* or *normalized* frequency f can be observed, a measure of the sampling rate of an analog signal to obtain a discrete signal, defined according to the following relationship:

$$f = \frac{F}{F_{es}} = \frac{T_{es}}{T}$$

where F is the analog signal frequency, T is the analog signal period, F_{es} is the sampling frequency, and $T_{es}=1/ F_{es}$ is the sampling period.

To avoid *aliasing* phenomena, which lead to signal distortion, signal sampling must follow Shannon's theorem:

$$F_{es} > F \Rightarrow f = \frac{F}{F_{es}} < 0,5$$

Thus, normalized frequencies always range from 0 to 0.5.

For a discrete signal, the number of samples over a period of the sampled signal N_{ep} can be determined as the inverse of the normalized frequency:

$$T = N_{ep} \cdot T_{es} \Rightarrow N_{ep} = \frac{T}{T_{es}} = \frac{1}{f}$$

3. Graphical representation of discrete signals in MATLAB

Discrete signals are plotted in MATLAB using the `stem()` function. This generates a graph similar to the `plot()` function, but displays individual points for each signal value, marking them on the graph according to their index.

The `stem` function syntax, `stem(x,n)` is used to plot a discrete signal $x[n]$ in which signal values are given by vector x and sampling times are given by vector n . The length of vector n must be equal to the length of vector x .

Example 1 Generating and representing the unit impulse signal

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{in the rest} \end{cases}$$

```
% Unit impulse signal generation
clear;
clc;
clear all
n = -10:20; % generation of a vector from -10 to 20
delta = [zeros(1,10) 1 zeros(1,20)]; % unit impulse generation
stem(n,delta) % graphic representation
grid on
xlabel('n');
ylabel('Amplitude');
title('Unit impulse (Dirac)');
axis([min(n) max(n) min(delta)-0.2 max(delta)+0.2]);
```

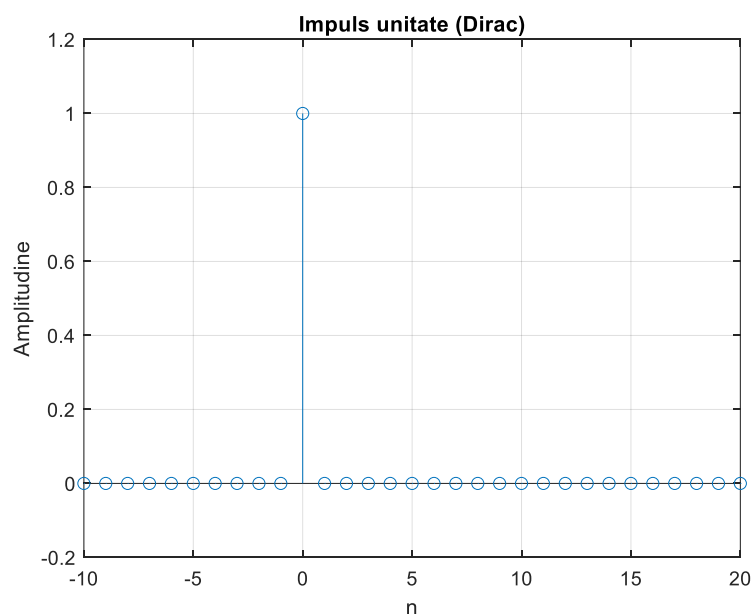


Figure 2. Representation of the unit impulse signal $\delta[n]$ in the range $n = -10:20$

Example 2 Generation and representation of a train of periodic pulses of amplitude A_l , period P and length $M \cdot P$:

$$s[n] = \sum_{l=0}^{M-1} A_l \delta[n - lP]$$

```
% Generation of a periodic impulse train
clear;
clc;
clear all
P = 5; M = 6;
d = [1;zeros(P-1,1)]; % generation of the generating impulse, of length P
y = d*ones(1,M);
train = y(:); % generation of impulse train, of length P*M
n = 0:M*P-1;
stem(n,train) % graphical representation
grid on
xlabel('n');
ylabel('Amplitude');
title('Unit impulse train');
axis([min(n)-2 max(n)+2 min(train)-0.2 max(train)+0.2]);
```

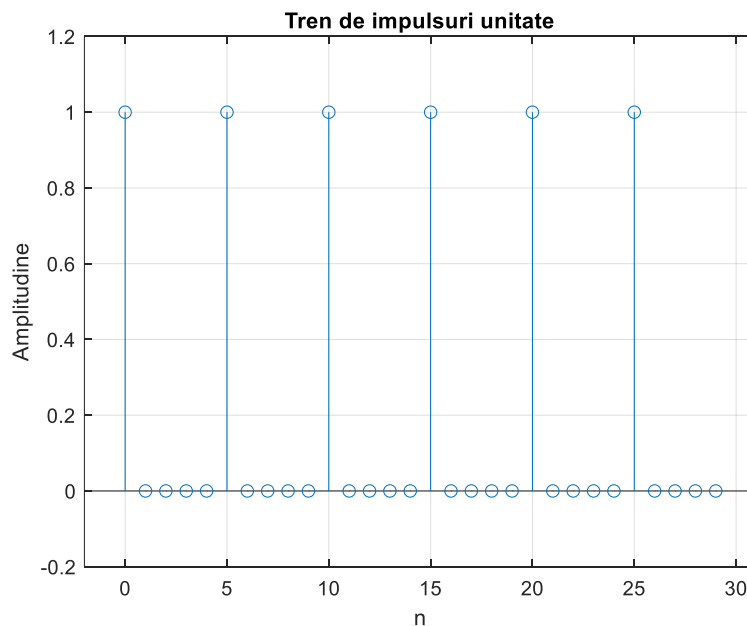


Figure 3. Representation of a train of periodic impulses of amplitude $A=1$, period $P=5$ and length $M \cdot P=30$

Example 3 Generating and representing the unit step signal:

$$u[n] = \begin{cases} 1, & n \in \mathbb{N} \\ 0, & \text{in the rest} \end{cases}$$

```

% Unit step signal generation
clear;
clc;
clear all
n = -10:20; % generating a vector from -10 to 20
u = [zeros(1,10) ones(1,21)]; % unit step generation
stem(n,u) % graphical representation
grid on
xlabel('n');
ylabel('Amplitude');
title('Unit step')
axis([min(n) max(n) min(u)-0.2 max(u)+0.2])

```

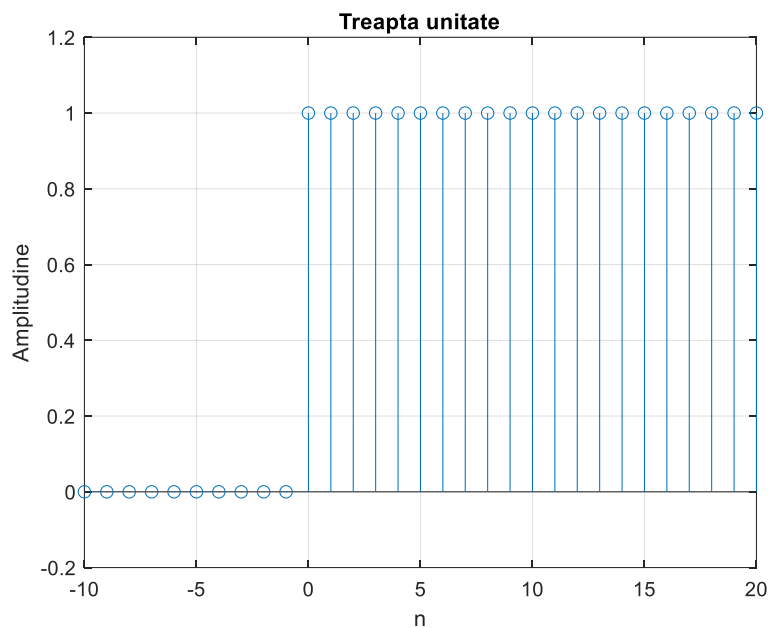


Figure 4. Unit step signal representation $u[n]$ in the range $n = -10:20$

Example 4 Generating and representing a true exponential sequence:

$$x[n] = a^n, n \in \mathbb{Z}, a \in \mathbb{R}$$

```

% Generating a true exponential sequence
clear;
clc;
clear all
n = 0:35; % generation of a vector from 0 to 35
a = 1.2;
K = 0.2;
x = K*a.^n; % generation of true exponential sequence
stem(n,x) % graphical representation
grid on
xlabel('n');
ylabel('Amplitude');
title('Real exponential sequence')

```

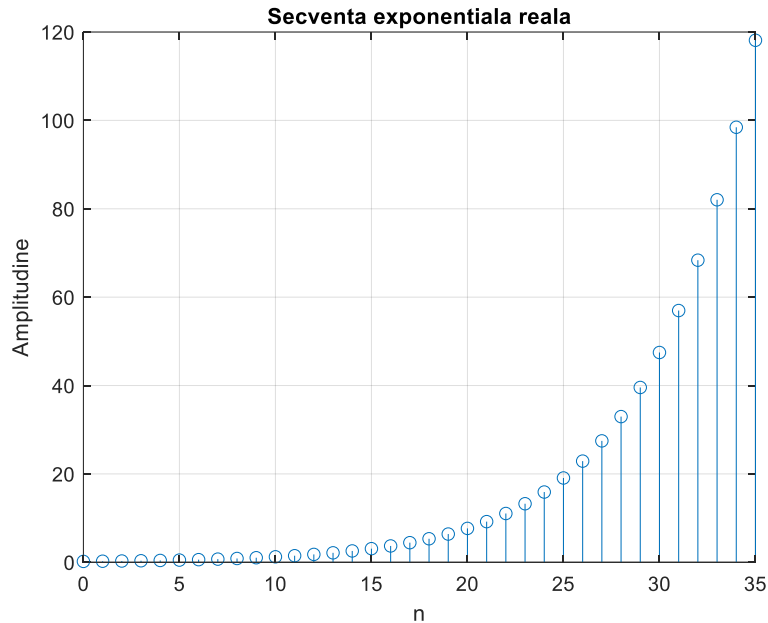


Figure 5. Representation of a real exponential sequence in the interval $n = 0:35$

Example 5 Generating and representing a complex exponential sequence:

$$x[n] = a^n, n \in \mathbb{Z}, a \in \mathbb{C}$$

Taking into account that the parameter a is complex, it can be written:

$$a = r \cdot e^{j\omega_0}$$

where r and ω_0 represent the modulus and phase of complex magnitude a , respectively. In this case, the signal $x[n]$ can be rewritten:

$$x[n] = r^n e^{j\omega_0 n} = r^n (\cos \omega_0 n + j \sin \omega_0 n)$$

where $x_R[n] = r^n \cos \omega_0 n$ and $x_I[n] = r^n \sin \omega_0 n$ represent the real and imaginary parts of the signal $x[n]$ respectively, which can be represented as a function of n .

```
% Generating a complex exponential sequence
clear;
clc;
clear all
n = 0:40; % generation of a vector from 0 to 40
c = -(1/12)+(pi/6)*1i;
K = 2;
x = K*exp(c*n); % generation of complex exponential sequence
figure(1) % graphic representation
subplot(2,1,1); stem(n,real(x));
grid on
xlabel('n'); ylabel('Amplitude');
title('The real part of the complex exponential sequence')
subplot(2,1,2); stem(n,imag(x));
grid on
xlabel('n'); ylabel('Amplitude');
title('The imaginary part of the complex exponential sequence')
```

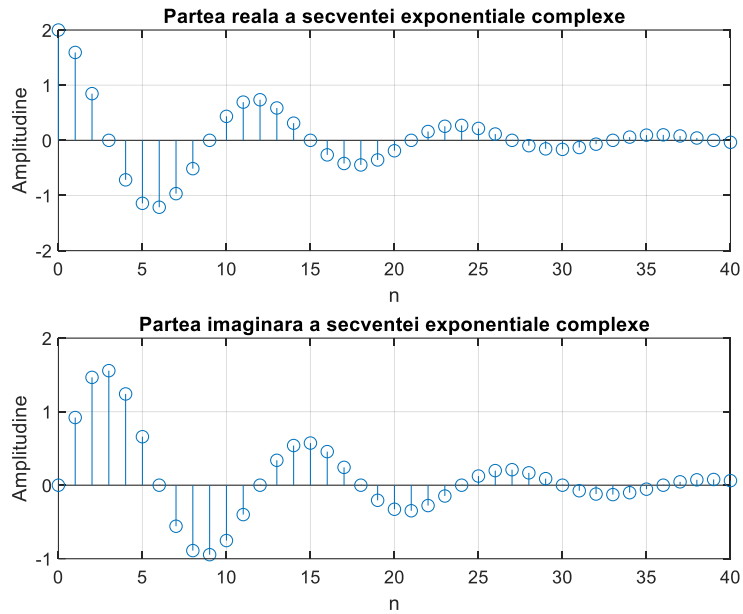


Figure 6. Representation of the real and imaginary parts of a complex exponential sequence in the range $n = 0:40$

Example 6 Sinusoidal signal generation and representation:

$$x[n] = A \cdot \sin(2 \cdot \pi \cdot f \cdot n), n \in \mathbb{Z}, f \in \mathbb{R}$$

```
% Sinusoidal signal generation
clear;
clc;
clear all
n = 0:40; % generation of a vector from 0 to 40
f = 0.1;
A = 1.5;
xsin = A*sin(2*pi*f*n); % sinusoidal signal generation
stem(n,xsin) % graphical representation
grid on
xlabel('n'); ylabel('Amplitudine'); title('Sinusoidal signal');
```

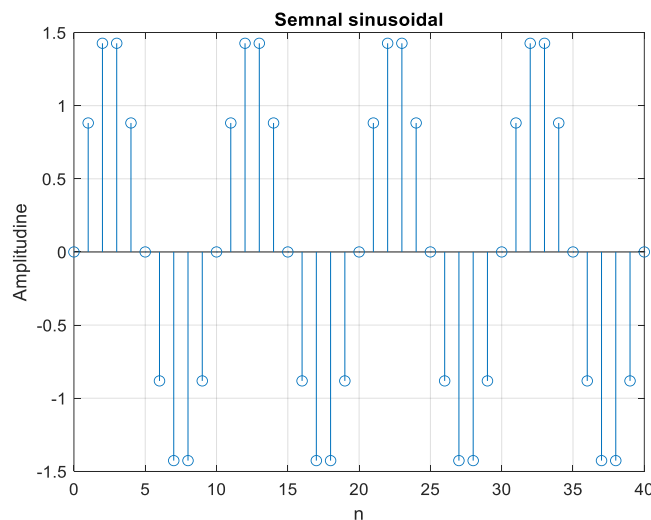


Figure 7. Representation of the sine signal in the range $n = 0:40$

Exercise 1

Generate and plot the following signals. The abscissa (n) should cover only the indicated area.

a. $x_a[n] = 0.7 \cdot \delta[n + 7]$ for $-15 \leq n \leq 15$

b. $x_b[n] = 1.5 \cdot u[n - 5]$ for $-5 \leq n \leq 20$

c. $x_c[n] = \sin\left(\frac{\pi}{17}n\right)$ for $-15 \leq n \leq 15$. What is the value of the discrete frequency?

d. $x_d[n] = \begin{cases} n + 1, & 0 \leq n \leq 3 \\ 0, & \text{in the rest} \end{cases}$ for $0 \leq n \leq 3$

e. $x_e[n] = \sum_{i=0}^{M-1} x_d[n - iP]$ for $M=5$, $P=4$ and $0 \leq n \leq 19$

Exercise 2

Generate and plot a rectangular signal and a sawtooth signal having 12 samples per period. Considering the n index in the interval $0 \leq n \leq 30$, how many periods are displayed for each signal?

Bibliography

1. V. Grigoras, D. Tărniceriu, *Prelucrarea numerică a semnalelor, Semnale si sisteme discrete*, Editura "Gh. Asachi", ISBN 973-9178-28-6, 128 pg., 1995
2. D. Tărniceriu, F. Beldianu, Indrumar de laborator *Prelucrarea numerică a semnalelor*, <http://telecom.etc.tuiasi.ro:81/pns/>, 150 pg., 2005.
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