Laplace transform using MATLAB

1. Introduction to the Laplace transform of continuous time signals

In the last semester, at Signal and system 1 – laboratory, it was used the series and the Fourier transform to determine the frequency domain characteristics, of certain types of continuous-time signals. For a signal to have Fourier series or transform it must be absolutely integrable. Thus, a function of the type$t \cdot \sigma(t)$ is not absolutely integrable, and therefore the Fourier transform cannot be calculated. In this case the Laplace transform can be applied. Thus, the Laplace transform can be considered as a generalization of the Fourier transform.

The Laplace transform (also called bilateral Laplace transform) for a given signal $x(t)$ is given by:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt,$$

where s is a complex number. The one-sided Laplace transform is defined:

$$X(s) = \int_{0}^{\infty} x(t)e^{-st}\,dt.$$

Thus, given a signal $x(t)$, the set of all complex numbers s for which the integral exists is called the Convergence Region. For example, for the step function, $\sigma(t)$, the region of convergence is given by $Real(s) > 0$.

The equation used to reconstruct the signal $x(t)$, knowing its Laplace transform, $X(s)$, is:

$$x(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} X(s)e^{st}\,ds$$

The integral is evaluated for $s = c + j\omega$ in the complex plane between the limits $c + j\infty$ and $c - j\infty$, where c is a real number for which s belongs to the region of convergence of $X(s)$. From a practical point of view, the integral is quite difficult to solve, so algebraic methods are used, such as: decomposition into simple fraction, using known pairs, $x(t) \underset{L}{\leftrightarrow} X(s)$ or determining the residues and poles of $X(s)$. A series of Laplace pairs can be found here http://ece-research.unm.edu/bsanthan/ece541/table_ME.pdf

The equation for the residuals is, for a signal $x(t), with\ x(t) = 0, t < 0$:

$$x(t) = \sum_{\substack{all\ plos\ of\ X(s) \\ from\ the\ left\ half\ plane}} Rez[X(s)e^{-st}]$$

Example 1

Using symbolic notation, determine the Laplace transform for the signal:

$$x(t) = \begin{cases} e^{-at}, t \geq 0 \\ 0, \text{otherwise} \end{cases}$$

```
%% MATLAB
%% Calculation of the Laplace transform
syms x a t;
x = exp(-a*t);
X = laplace(x) % function that generates the Laplace transform for
symbolic writing
```

The result obtained:

```
X =

1/(a + s)
```

Assignment 1

Using symbolic notation, determine the Laplace transform for the signals:

$$x_1(t) = \begin{cases} \cos(\omega t), t \geq 0 \\ 0, \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} \sin(\omega t), t \geq 0 \\ 0, \text{otherwise} \end{cases}$$

$$x_3(t) = \begin{cases} \frac{t^4}{4!}, t \geq 0 \\ 0, \text{otherwise} \end{cases}$$

Example 2

Using symbolic notation, determine the inverse Laplace transform for the signal:

$$X(s) = \frac{s+2}{s^3 + 4s^2 + 3s}$$

```
%% MATLAB
%% Calculation of the inverse Laplace transform
syms X s x ; % the symbols
X = (s+2)/(s^3+4*s^2+3*s); % Laplace transform
x = ilaplace(X) % inverse transform
```

The result obtained:

```
x =

2/3 - exp(-3*t)/6 - exp(-t)/2
```

Example 3

Determine the parameters of the signal $x(t)$ knowing that the Laplace transform is:

$$X(s) = \frac{s+2}{s^3+4s^2+3s}$$

```
%% Calculation of the Laplace transform using numerical methods
numaratorul = [1 2]; % numerator coefficients
numitorul = [1 4 3 0]; % denominator coefficients
[r,p] = residue(numaratorul,numitorul); % calculation of residues
(r) and poles (p)
```

The result obtained:

```
r =

  -0.1667
  -0.5000
   0.6667


p =

  -3
  -1
   0
```

Using the results obtained for r and p yields the inverse signal $x(t)$

$$x(t) = 0.66667 - 0,5 \cdot e^{-t} - 0,16667 \cdot e^{-3t}, t \geq 0$$

Assignment 2

Determine the signal parameters $x(t)$ (symbolic and numerical) knowing that the Laplace transform is:

$$X(s) = \frac{5s - 1}{s^3 - 3s - 2}$$

Assignment 3

A. Determine the Laplace transform for the signals:

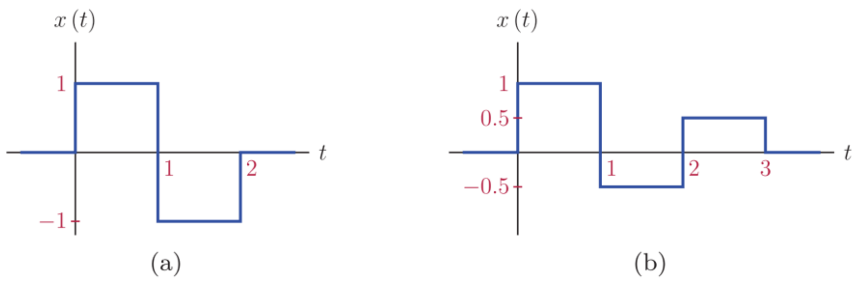
$$x_1(t) = \begin{cases} e^{-2t}, t \geq 0 \\ 0, \text{otherwise} \end{cases}$$

$$x_2(t) = \delta\,(t - 7)$$

$$x_3(t) = \sigma\,(t - 7)$$

$$x_4(t) = \begin{cases} 1, 0 < t < 1 \\ -4, 1 < t < 2 \\ 0, \text{otherwise} \end{cases}$$

B. Determine the Laplace transform for the signals from the below figure:



C. Determine the signal whose Laplace transform is given by the following expressions. Consider the signal is causal.

$$X_1(s) = \frac{1 - e^{-s}}{s + 1}$$

$$X_2(s) = \frac{s + 1}{(s + 2)(s + 3)}$$

Bibliography

1. Mateescu, Adelaida, Dumitriu, N., Stanciu, L., **Semnale, circuite și sisteme**, Teora, București, 2001.
2. Răzvan Eusebiu Crăciunescu, Valentin Adrian Niţă, Radu Alexandru Badea, Semnale şi programare : de la teorie la aplicaţii folosind MATLAB/Octave, indrumar de laborator Editura Politehnica Press, Bucureşti 2022, ISBN (print) 978-606-9608-00-5