First Laboratory

PERIODIC SIGNALS

1.1. The objective of the laboratory

In this laboratory will be studied the spectral analysis of the periodic signals. To achieve this objective, the amplitude spectrum of the sinusoidal signal, of the rectangular periodic signal with different duty cycles and of the triangular signal will be measured. For the rectangular signal (with different duty cycles) and for the triangular signal, the power will be determined using the experimental data and it will be compared with the power obtained using the time-domain representation of those signals.

1.2. Theoretical aspects

A periodic signal $x(t)$ is mathematically represented as a periodic function, in other words, for which there is a real nonzero number T, called *period*, such that the following equality is achieved:

$$x(t+T) = x(t), \quad \forall t \in \mathrm{R}\,. \tag{1}$$

If T is period and ensures the fulfilment of equation (1), then any multiple, kT, where $k \in \mathbf{Z}^*$, it is also a period for the signal. The lowest, strictly positive value of the period is called the *main period* (or *repetition period*) of the signal.

The usual signals encountered in practice have a moment of occurrence and a moment of extinction, in other words they can fulfil equation (1) only on a finite length of the time axis, which means that rigorous periodic signals do not exist in practice. However, in some situations, it is useful to model a finite-length signal, having on its existence a periodic variation, using a periodic function that fulfils (1) on the all real axis. This modelling does not lead to errors if the duration of the signal is much higher than the repetition period and than the duration of the transient regimes occurring in the circuit when applying or suppressing the signal and, moreover, if these transient regimes are not in area of interest.

A periodic signal $x(t)$, by T period, can be developed in Fourier series if the Dirichlet conditions are satisfied.

The formulas of the Fourier series and the relations used for calculation of the coefficients are given in Table 1.

Table 1 The Fourier series expressions of an analogic periodic signal

THE SERIES TYPE	**ANALITIC REPRESENTATION**	**RELATIONS FOR THE COEFICIENTS**
Exponential (complex)	$x(t)=\sum_{k=-\infty}^{\infty} A_{kc} e^{jk\omega_0 t}$	$A_{kc}=\frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jk\omega_0 t}dt$
Trigonometric	$x(t)=c_0+\sum_{k=1}^{\infty} c_k \cos k\omega_0 t + \sum_{k=1}^{\infty} s_k \sin k\omega_0 t$	$c_0=\frac{1}{T}\int_{t_0}^{t_0+T} x(t)dt$ $c_k=\frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos k\omega_0 t dt$ $s_k=\frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin k\omega_0 t dt$
Cosine (Harmonic)	$x(t)=A_0+\sum_{k=1}^{\infty} A_k \cos(k\omega_0 t+\varphi_k)$	$A_0=c_0$ $A_k=\sqrt{c_k^{\,2}+s_k^{\,2}}=2\lvert A_{kc}\rvert$ $\varphi_k=-arctg\frac{s_k}{c_k}=\arg\{A_{kc}\}$

$\omega_0=\frac{2\pi}{T}=2\pi f_0$ represents the *fundamental angular frequency* (pulsation), and f_0 is the *fundamental frequency*; which is also called the *repetition frequency* of the periodic signal.

The choice regarding the limits of integration in the process of evaluating the coefficients from the Fourier series is arbitrary, it is made in such a way as to simplify the calculations; it is very important that the integration is made over a period (from $-T/2$ to $+T/2$, from 0 to T, etc.).

The Exponential Fourier Series provides a decomposition of the periodic signal into a sum of elementary exponential components $e^{jk\omega_0 t}$, physically unachievable. Its use is very convenient in the problems in which is required to determine the response of circuits to periodic signals.

Practically (experimentally), we are interested in the Harmonic Fourier Series (HFA). The following details apply only to this development. This decomposes the signal into a sum of cosine signals (hereinafter referred to as *components*), whose frequencies are equal to multiples of the repetition frequency of the periodic signal. These components are also known as *harmonics.* The *component* situated on the zero frequency is called *DC offset* (*the continuous component*), the component at the f_0 frequency is *the fundamental component* (often called "the *fundamental*", *"the first order harmonic"* or *"the repetition frequency"*), and the components situated at the frequencies $kf_0\left(k \in \mathrm{N}, k \geq 2\right)$ are *the harmonic components* ("the *harmonics of* k *order*"). The assembly of these components forms the *spectrum of the signal*. Note that, in the case of periodic signals, the *spectrum is discreet*, with components only at certain frequencies, since periodic signals can be represented by *discrete sums* of elemental signals, as shown in Figure 1.

The characterization of the periodic signals, in the frequency domain, is made using *the amplitude spectrum* and *the phase spectrum*, meaning a graphical representation of the amplitude-frequency dependencies and, respectively, of the initial phase-frequency dependencies of the components. For this purpose, to each component from the development is assigned a straight segment (*spectral line*) in the two spectra, *located at the frequency of the component* and having the amplitude of the segment proportional to the amplitude or phase of the component.

The sign $"\mathrm{x}"$ from the amplitude spectrum shows that the amplitudes are null and in the phase spectrum it shows that, in those cases, the notion of the initial phase is meaningless or not determined; the component at $f = 4f_0$ does not exist (has zero amplitude, so there is no need to determine the initial phase of a zero amplitude signal). The case of periodic signals with DC offset is not discussed in this paper.

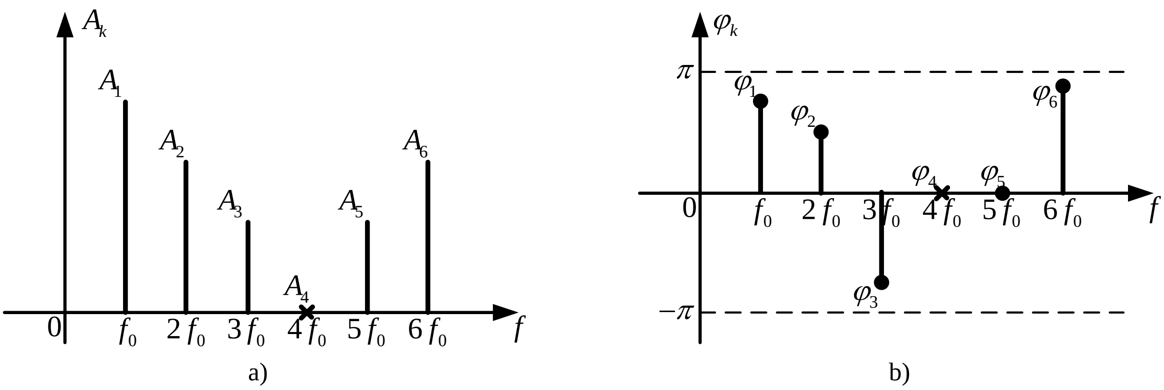

Figure 1. a) Amplitude spectrum diagram; b) Phase spectrum diagram

It is noted that, it is sufficient to know the amplitude and phase spectra to have a complete determination of the signal.

Theoretically, the signal spectrum ranges from zero frequency, $f = 0$, to the infinite frequency, $f = +\infty$; in practice, the components situated at very high frequencies are negligible because of their small amplitude values, so for signals used in practice, the bandwidth of the signal is finite, meaning, the spectrum is limited. The decreasing in amplitude of the components when frequency increases is even faster as the signal is smoother (the mathematical function used for the representation is derivative as many times as possible). The spectral truncation depends on the requirements imposed by the type of communication which is using the signal. Therefore, the spectral analysis of a signal allows us to determine the *effective bandwidth occupied* by that signal.

It is called " *effective bandwidth*" the bandwidth occupied by important components of the considered application. The effective bandwidth depends on the threshold value below which the components from the amplitude spectrum can be considered as negligible. When the threshold value increases, the effective bandwidth decreases; the choice of the negligence threshold is made according to a criterion established on practical considerations for each application. If the effective bandwidth of the signal is known, it is possible to determine the bandwidth in which the circuits that process the signal function correctly.

A. The harmonic signal

The analytical expression of a harmonic signal is:

$$x(t) = A \cdot \cos(\omega_0 t + \varphi), \tag{2}$$

and the graphical representation of the amplitude spectrum is shown in Figure 2, where $f_0 = \frac{1}{T} = \frac{\omega_0}{2\pi}$.

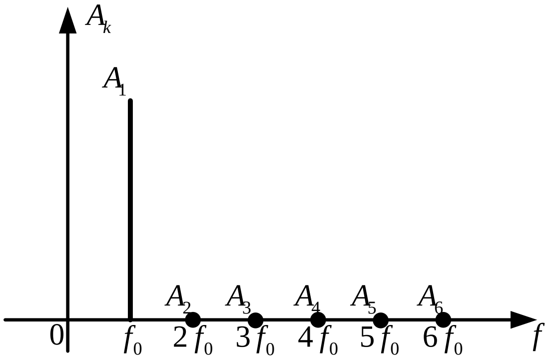

Figure 2. The diagram of the amplitude spectrum of a harmonic signal

A pure harmonic signal has a distortion factor of 0% (does not have harmonics with an order higher than 1). The real signal obtained from the function generator used in the laboratory is not perfectly sinusoidal, which implies the presence of non-zero spectral components for frequencies that are multiples of the fundamental frequency. We are interested in finding out how much the signal from the function generator differs from the pure harmonic signal or, in other words, how distorted (modified) is the generated harmonic signal; the distortions appear due to inherent nonlinearities in the generator circuits. To measure these distortions the harmonic distortion factor was introduced δ, and it is defined as follows:

$$\delta = \frac{\sqrt{A_2^2 + A_3^2 + ...}}{A_1} = \sqrt{\sum_k 10^{\frac{n_k - n_1}{10}}}, \tag{3}$$

where, $n_k = 20\lg\frac{A_k}{U_r}$, and the reference voltage, U_r, will be explained later.

δ must be as small as possible, close to zero.

B. The triangular signal

Using Table 1, the Harmonic Fourier Series of the symmetrical triangular periodic signal is calculated, with repetition frequency f_0 (Figure 3.a):

$$x(t) = \sum_{k=1}^{\infty} 2E \mathrm{sinc}^2 \frac{k\pi}{2} \cos k\omega_0 t = \sum_{n=0}^{\infty} \frac{8E}{\pi^2 (2n+1)^2} \cos(2n+1)\omega_0 t . \tag{4}$$

From (4), the amplitude of the spectral components is identified:

$$A_k = \begin{cases} \dfrac{8E}{\pi^2 k^2} & , k \text{ odd} \\ 0 & , k \text{ even} \end{cases} . \tag{5}$$

The graphical representation of the amplitude spectrum of the signal from Figure 3.a) is given in Figure 3.b), and the amplitude spectrum normalized to the amplitude of the fundamental component is found in Figure 3.c).

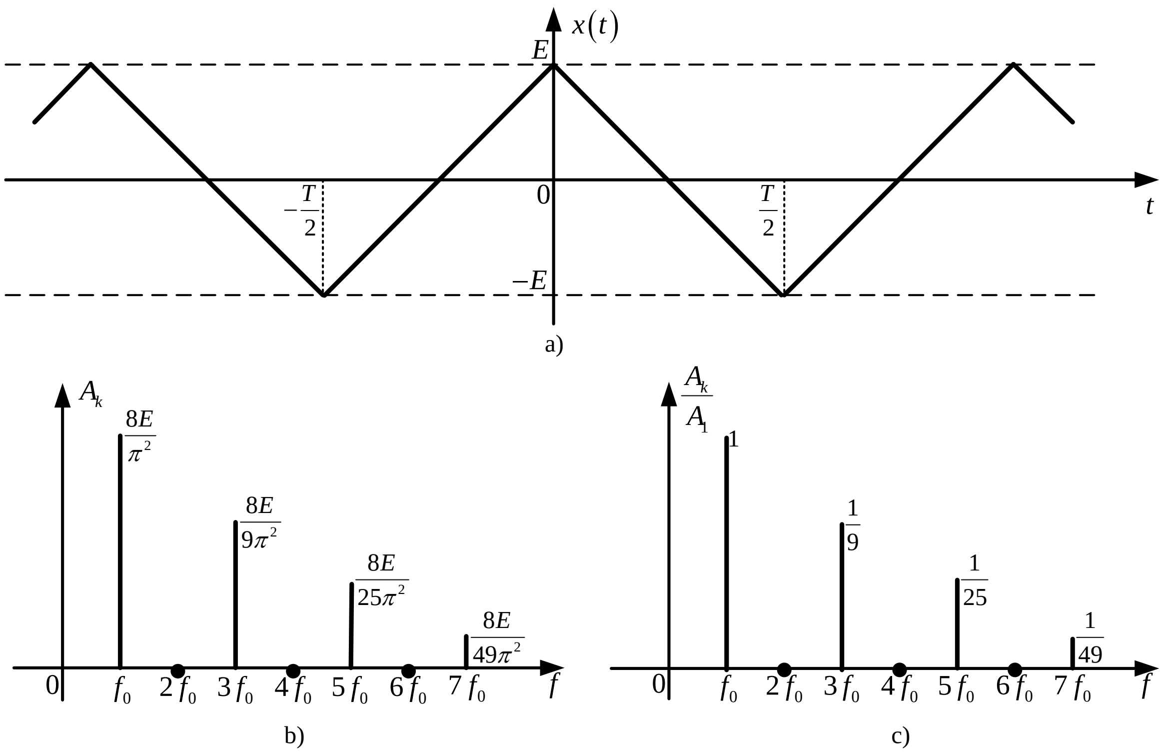

Figure 3. a) Time representation of the triangular signal; b) Amplitude spectrum for the triangular signal; c) Normalized amplitude spectrum for the triangular signal

The power of the symmetrical triangular signal, dissipated on a 1Ω resistor, can be calculated based on experimental data according to the relationship:

$$P_e = \sum_{k=1}^{k_M} \frac{A_k^2}{2}, \tag{6}$$

where k_M is the number of harmonics that are found in the amplitude spectrum.

If the time domain representation of the signal is used, the power of the triangular signal on a 1Ω resistor, is calculated as follows:

$$P_t = \frac{1}{T}\int_{(T)} |x(t)|^2 \, dt = \frac{E^2}{3} = X_{ef}^2, \tag{7}$$

where X_{ef} is the effective value of the analysed signal.

C. The rectangular signal

The graphical representation of a rectangular signal is shown in Figure 4. The signal is even, so the amplitudes s_k, of the trigonometric series are null and $A_k = |c_k|$.

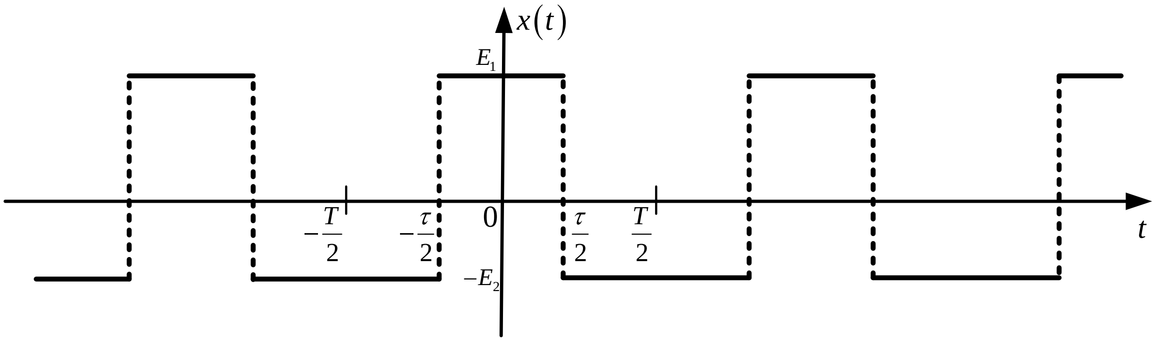

Figure 4. The graphic representation of the rectangular signal without DC offset and duty cycle $\frac{\tau}{T}$

From the point of view of the amplitude spectrum, the signal parity does not matter because the movement along the time axis only results in the modification of the phase spectrum, φ_k, the amplitude spectrum, A_k, will be the same.

Using the relationships from Table 1 the expression of the Harmonic Fourier Series is:

$$x(t)=\sum_{k=1}^{\infty}\frac{2(E_1+E_2)}{k\pi}\sin(k\pi\frac{\tau}{T})\cos(k\omega_0 t). \tag{8}$$

From (8) the expression of A_k becomes:

$$A_k=2(E_1+E_2)\frac{\tau}{T}\left|\frac{\sin(k\pi\frac{\tau}{T})}{k\pi\frac{\tau}{T}}\right|=2(E_1+E_2)\frac{\tau}{T}\left|\mathrm{sinc}(k\pi\frac{\tau}{T})\right|, \tag{9}$$

which highlights the fact that the amplitudes of the signal harmonics decrease after a $\left|\mathrm{sinc}x\right|=\left|\frac{\sin x}{x}\right|$ shape envelope. They also show their proportionality with the amplitude E_1+E_2 of the periodic rectangular signal as well with the ratio τ/T called *duty cycle*.

When the rectangular signal has a duty cycle $\frac{\tau}{T}=\frac{1}{2}$, the amplitudes E_1 and E_2 are equal in module ($|E_1|=|E_2|$). If the duty cycle decreases, $|E_1|$ increases and $|E_2|$ decreases to obtain a zero DC offset. Changing the duty cycle without changing the amplitudes $|E_1|$ and $|E_2|$ leads to changing the DC offset.

The harmonics for which the following condition is met $k\pi\frac{\tau}{T}=p\pi$ (meaning $k=p\frac{T}{\tau}$), p being an integer, have zero amplitudes. For example, for $\frac{\tau}{T}=\frac{1}{2}$ and $\frac{\tau}{T}=\frac{1}{10}$, the even harmonics will be null, $k=2p$, respectively harmonics of $k=10p$ order.

Figure 5 shows the amplitude spectrum of the signal $x(t)$ from Figure 4, maintaining the T period constant, and the τ width of the impulse equal to $T/2$, respectively $T/10$.

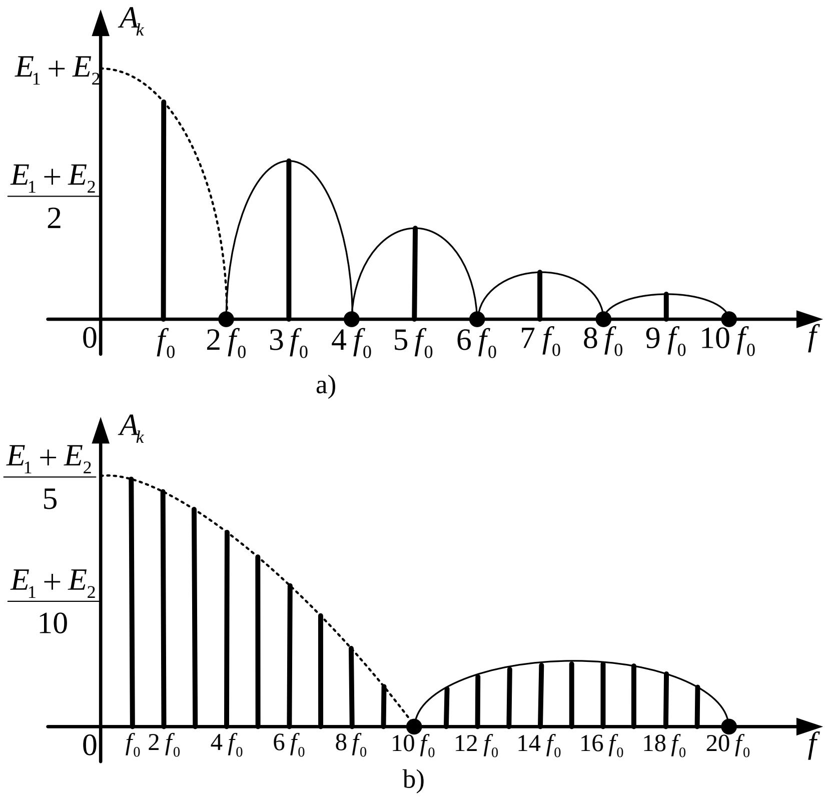

Figure 5. a) Amplitude spectrum of the rectangular signal with duty cycle $\tau/T = 1/2$
b) Amplitude spectrum of the rectangular signal with duty cycle $\tau/T = 1/10$

The normalized amplitude spectrum is obtained by the normalization of A_k at the fundamental amplitude value A_1.

$$\left|\frac{A_k}{A_1}\right| = \frac{1}{k} \cdot \frac{\left|\sin k\pi \frac{\tau}{T}\right|}{\sin \pi \frac{\tau}{T}}. \tag{10}$$

This ratio highlights a decrease in amplitude of the harmonics compared to the fundamental.

So, for a $\tau/T = 1/2$ the relationship (10) becomes:

$$\left|\frac{A_k}{A_1}\right| = \frac{1}{k}\cdot\frac{\left|\sin\frac{k\pi}{2}\right|}{\sin\frac{\pi}{2}} = \begin{cases}\frac{1}{k} & ,k \text{ odd}\\ 0 & ,k \text{ even}\end{cases} . \tag{11}$$

The graphical representation of the normalized amplitude spectrum, in this case, is given in Figure 6.

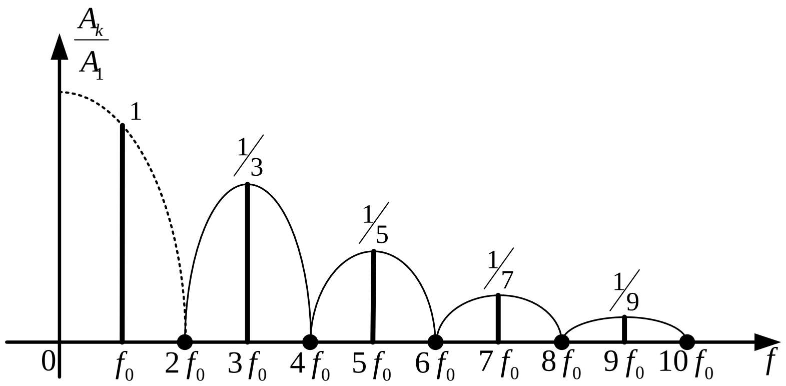

Figure 6. Graphical representation of the normalized amplitude spectrum of a rectangular periodic signal with duty cycle $\tau/T = 1/2$

The power of the rectangular signal, dissipated on a 1Ω resistor, can be calculated based on experimental data according to the relationship:

$$P_e = \sum_{k=1}^{k_M} \frac{A_k^2}{2}. \tag{12}$$

If the time domain representation of the signal (Figure 4) is used, the power of the rectangular signal strength on a 1Ω, is calculated using the relation:

$$P_t = \frac{1}{T}\int_{(T)} |x(t)|^2\, dt = \frac{\tau}{T}\left(E_1^2 - E_2^2\right) + E_2^2. \tag{13}$$

1.3. The experimental part of the laboratory

The block diagram of the assembly is shown in Figure 7.

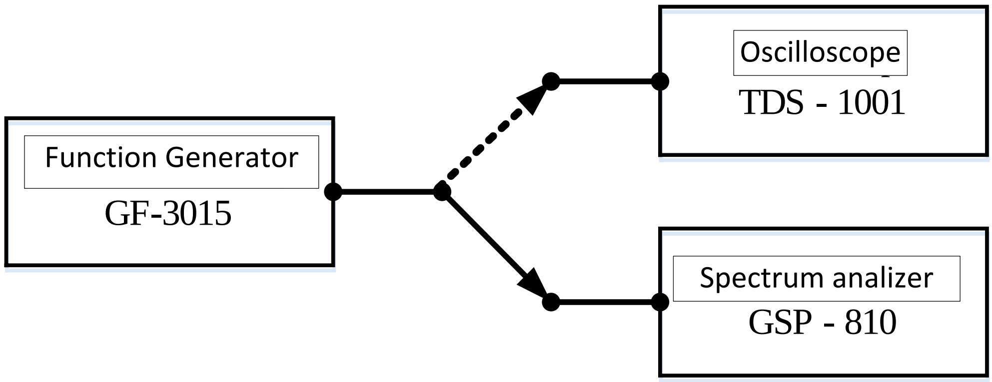

Figure 7. The block diagram of the assembly for periodic signals

A) The assembly from Figure 7 is realized

When making measurements, reading them and interpreting the measured values, take into account that V_{rms} is the effective value of the voltage expressed in volts (rms = root mean square).

For the devices used in the laboratory, dBm is the unit of measure for the voltage expressed in decibels having the reference voltage, the voltage corresponds to a power of 1mW on a resistance of 50 Ω. So, for a $P = 1\,\text{mW}$ and a $R = 50\,\Omega$, the actual reference voltage is obtained as follows:

$$U_{r,ef} = \frac{U_r}{\sqrt{2}} = \sqrt{PR} = \sqrt{10^{-3} \cdot 50} = 0,2236\ V\,.$$

This voltage is used when between the generator and spectrum analyzer is a perfect matching. Otherwise, for measurements in frequency domain using the analyzer, the reference voltage is determined by applying from the function generator a 200 kHz sinusoidal signal whose amplitude is adjusted such that the amplitude of the fundamental component on the spectrum analyzer is 0 dBm.

Then the sinusoidal signal from the function generator is visualized with the oscilloscope. The peak value of the measured reference voltage $U_{r,m}$ will be the peak amplitude E_0 of the sinusoidal signal divided by 2. This is happening because the function generator and the spectrum analyzer used have an input impedance of 50 Ω, which is much lower than the input impedance of 1 M of the oscilloscope. As a result, the measured effective reference voltage will be:

$$U_{r,m,ef} = \frac{E_0}{2\sqrt{2}} = \frac{U_{r.m}}{\sqrt{2}}$$

It is reminded that for a voltage U, its level in dB is:

$$n = 20\lg\frac{U}{U_r} \quad [\text{dB}], \tag{17}$$

where U_r is the reference voltage.

B) Theoretical and experimental spectral analysis of the rectangular periodic signal with duty $\tau/T = 1/2$.

At the function generator set the waveform (FUNC button) to be rectangular, the frequency (FREQ button) $f_0 = 200\,\text{kHz}$, the duty (DUTY button) $\tau/T = 1/2$ (i.e. 50%) and the amplitude (AMPL button) E of the rectangular signal such that the level of the fundamental harmonic (the spectral component placed at the signal frequency, in this case $200\,\text{kHz}$) measured with the spectrum analyzer to be 0 dBm. To avoid setting errors, set the reference level to 10 dBm - REF LVL button. To measure the harmonics, the centre frequency of the analyzer ("CENTER" button) is fixed at $1\,\text{MHz}$ and SPAN at $200\,\text{kHz/div}$ (in this way you can see more harmonics on the analyzer screen, starting with the fundamental harmonic). One of the markers is adjusted to the harmonic frequency of the component which is to be measured and read the value indicated in dBm (for example, it is desired to measure the second harmonic. The cursor is adjusted to $0{,}400\text{ MHz}$ and read the value indicated in dBm. Because the second harmonic is measured, the measured value will represent A_2 expressed in dBm. These notations are used in the calculations below. When the cursor indicates HIGH or LOW it means that it has been set a higher or a lower value of the maximum frequency, respectively, of the minimum frequency that can be viewed on the analyzer screen, which means that the analyzer centre frequency must be changed to a higher value, respectively smaller.

Measure the level in dBm of the first 20 harmonics (A_1, A_2 etc.) and write them down separately in Table 2. In Table 2 we have:

- k - the order of harmonics,

- $f_k\left[\mathrm{MHz}\right]$ - the harmonic frequency of k order,
- For the experimental part:

  - Using markers A_1, A_2, etc. were measured in dBm;

  - Write in the table $\left.\frac{A_k}{A_1}\right|_{\mathrm{exp}}\left[\mathrm{dB}\right]=A_k\left[\mathrm{dBm}\right]-A_1\left[\mathrm{dBm}\right]$.

D. For the theoretical part (calculated at home):

- $\left.\frac{A_k}{A_1}\right|_{\mathrm{th.}}$ - is calculated with equation (11);

- $\left.\frac{A_k}{A_1}\right|_{\mathrm{th.}}\left[\mathrm{dB}\right]=20\lg\frac{A_k}{A_1}$.

Table 2 Spectral analysis of the rectangular periodic signal with $\tau/T=1/2$

k	1	2	3	4	5	...	19	20
$f_k\left[\mathrm{MHz}\right]$	0,2	0,4	0,6	0,8	1	...	3,8	4
$\left.\frac{A_k}{A_1}\right\|_{\mathrm{th}}$								
$\left.\frac{A_k}{A_1}\right\|_{\mathrm{th}}\left[\mathrm{dB}\right]$								
$\left.\frac{A_k}{A_1}\right\|_{\mathrm{expe}}\left[\mathrm{dB}\right]$								
$\left.\frac{A_k}{A_1}\right\|_{\mathrm{expe}}$								

The values E_{01} and $-E_{02}$ of the studied signal are measured using the oscilloscope: press the Autoset button, then press the Measure button, select one of the 5 simultaneous measurements available by pressing one of the 5 buttons situated on the right side of the screen. Select the source of the channel to which the studied signal was connected, and the type of measurement Max (for E_{01}), respectively, Min (for $-E_{02}$).

C) Theoretical and experimental spectral analysis of the rectangular periodic signal with duty $\tau/T = 1/4$.

The theoretical and experimental spectral analysis of the same rectangular periodic signal with $f_0 = 200\,\mathrm{kHz}$, but with $\tau/T = 1/4$ (i.e. 25%, change made by pressing the DUTY button and adjusted by using the rotary knob). Pay attention to the E amplitude (AMPL button). After changing the duty, E amplitude is set such that the level of the fundamental harmonic is equal to 0 dBm, similar to the experiment from point B. The calculations are like those from the previous point.

Table 3 Spectral analysis of the rectangular periodic signal with $\tau/T = 1/4$

k	1	2	3	4	5	...	19	20
$f_k\,[\mathrm{MHz}]$	0,2	0,4	0,6	0,8	1	...	3,8	4
$\left.\frac{A_k}{A_1}\right\|_{\mathrm{th}}$								
$\left.\frac{A_k}{A_1}\right\|_{\mathrm{th}}$ [dB]								
$\left.\frac{A_k}{A_1}\right\|_{\mathrm{expe}}$ [dB]								
$\left.\frac{A_k}{A_1}\right\|_{\mathrm{expe}}$								

The values E_{01} and $-E_{02}$ of the studied signal are measured as described/mentioned /explained at the previous point.

D) The determination of the bandwidth occupied by the rectangular periodic signals studied above.

It is taken into account that in the bandwidth are included all the components that have amplitudes higher than 1% of the fundamental amplitude, meaning $0{,}01 \cdot A_1[\mathrm{V}]$. Firstly, the amplitude of the fundamental is set at 0 dBm, like in the above experiments. That is, $20\lg\frac{A_1}{U_r}=0\,[\mathrm{dBm}]$. So, $0{,}01\cdot A_1\,[\mathrm{V}]$ expressed in dBm will be:

$$20\lg\frac{0{,}01A_1}{U_r}=20\lg 0{,}01+20\lg\frac{A_1}{U_r}=-40+0=-40\,[\mathrm{dBm}].$$

Therefore, we will be looking for all the components that have amplitudes higher than $-40\,\mathrm{dBm}$. Taking in consideration that in the case of a rectangular signal, with duty cycle 50%, the harmonics of even order are very small, practically zero, we will be out of the bandwidth only when minimum 3 consecutive harmonics have the level less than $-40\,\mathrm{dBm}$. In the case of a rectangular signal, with duty cycle 25%, the harmonics of $k=4p,\ p\in\mathrm{N}^*$ order are very small (practically zero), so we are out of the bandwidth when minimum 5 consecutive harmonics have the level less than $-40\,\mathrm{dBm}$. For performing experiments only at the limit of the bandwidth we proceed as follows: at the function generator it is set one of the above periodic signals. Pay attention to the correct adjustment of the E (AMPL button) after changing the duty cycle (DUTY button) such that the level of the fundamental is $0\,\mathrm{dBm}$, similar to the experiment at point B. Adjust the reference level of the spectral analyzer to $10\,\mathrm{dBm}$ (REF button LVL), that is the maximum measurable level. If all the adjustments were made correctly, the fundamental harmonic level is near a horizontal black line drawn from the spectrum analyzer screen (see Figure 9), line indicating a level equal to $0\,\mathrm{dBm}$. A vertical division of the screen has $10\,\mathrm{dBm}$, so to identify the horizontal line of $-40\,\mathrm{dBm}$ we have to identify the fourth horizontal black line below that of the $0\,\mathrm{dBm}$ horizontal line . Then, it is increased the value indicated by CENTER until the harmonics are close to $-40\,\mathrm{dBm}$ (see Figure 10).

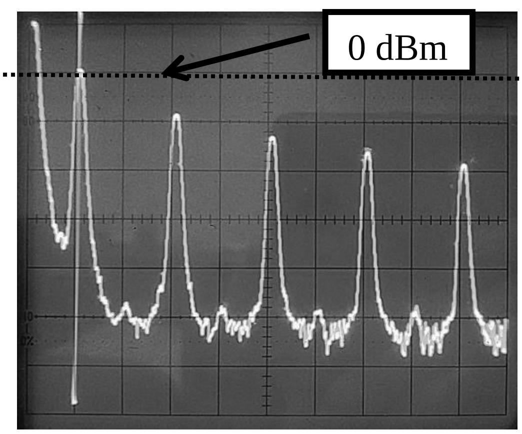

Figure 9. The spectrum of the rectangular signal with duty cycle 50% (low frequencies).

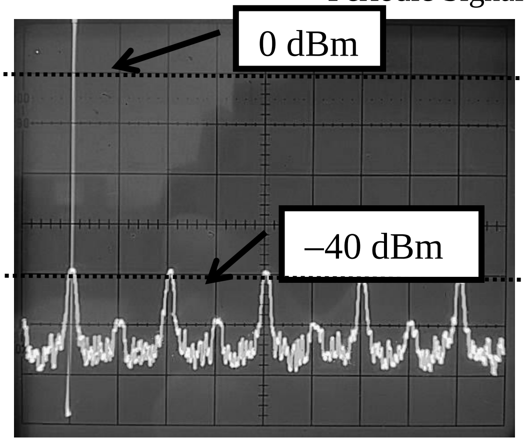

Figure 10. The spectrum of the rectangular signal with duty cycle 50% (high frequencies).

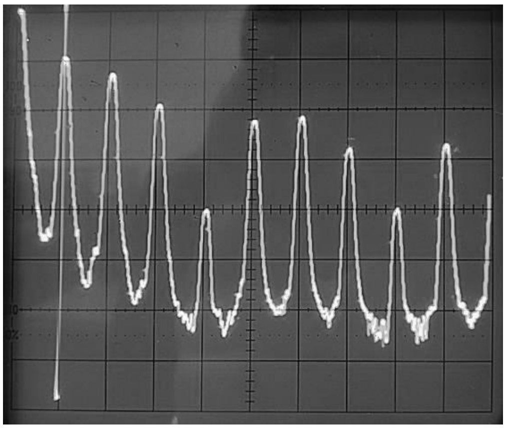

Figure 11. The spectrum of the rectangular signal with duty cycle 25% (low frequencies).

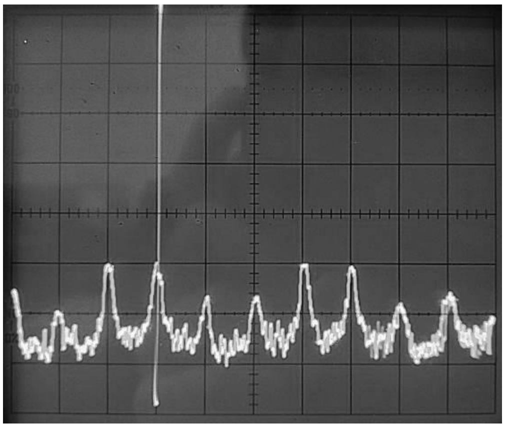

Figure 12. The spectrum of the rectangular signal with duty cycle 50% (high frequencies).

Place one of the cursors on the screen, set the frequency value equal to the value indicated by CENTER. Considering that the generator is set to a signal of $200\,\text{kHz}$, its harmonics are located on multiple of $200\,\text{kHz}$. The level of the harmonics, that can be viewed on the spectral analyzer screen, is measured from left to right. Measurements shall be made until at least 3 (duty cycle 50%) or 5 (duty cycle 25%) consecutive components are found, below the level of $-40\,\text{dBm}$. The last component indicating a higher value of $-40\,\text{dBm}$ is the last component in the spectrum, situated on frequency $k \cdot 200$ kHz. Since the signals are in the base band, the signal bandwidth is $B = k \cdot 200$ kHz . If all of the components on the screen are higher than $-40\,\text{dBm}$, the CENTER value is increased and the level of the harmonics is still measured. If the components have lower values than $-40\,\text{dBm}$, the value indicated by CENTER is decreased.

E) The theoretical and experimental spectral analysis of symmetrical triangular periodic signal.

The waveform generated by the function generator is changed to obtain a triangular signal (FUNC button). Its frequency is set to $f_0 = 200\,\text{kHz}$, signal symmetry at 50% (DUTY button) and amplitude of the triangular signal E so that the fundamental level measured with the spectral analyzer is $0\,\text{dBm}$. For the triangular signal the first 12 spectral components are measured and under these conditions, it is determined the bandwidth occupied by the triangular signal following the steps described above. The experimental results are then completed into a table similar to Table 2, and the theoretical ones will be completed at home.

The E_0 amplitude for the studied signal is measured using the oscilloscope following these steps: press the Autoset button, press the Measure button, select one of the 5 simultaneous measurements available by pressing a button from the 5 ones situated on the right of the screen. The channel to which the studied signal was connected is selected as the source, and Max as the type of measurement.

F) Determining the distortion factor.

A sinusoidal signal (FUNC button), generated by the function generator, having a frequency of $f_0 = 200\,\text{kHz}$ and the level of the fundamental equal to 0 dBm, is applied at the input of the spectrum analyzer. Measure the level of the first 10 spectral components and calculate the distortion factor using the relationship (3). Repeat the measurements for a sinusoidal signal produced by the function generator, having a fundamental frequency of $f_0 = 200\,\text{kHz}$ and a reference level of 15 dBm.

G) Repeat point F) for a triangular signal with $f_0 = 10\,\text{kHz}$ and for frequency domain measurements using the TDS 1001 oscilloscope.

The TDS 1001 oscilloscope can also be used as a spectral analyzer. In order to enter in the spectral analysis mode, a signal is connected to the input of one of the two channels, Autoset button is pressed, then the Math menu button is pressed, FFT (Fast Fourier Transform) is selected and at Source is selected the channel on which the signal is connected. Use the rotary knob sec / div to set span to 12.5 kHz / div.

For measurements in the frequency domain the oscilloscope is used. The reference voltage is determined by applying to the function generator a sinusoidal signal with the frequency $10\,\text{kHz}$ and whose amplitude is adjusted so that the amplitude of the fundamental component, viewed on the oscilloscope, used as a spectrum analyzer to be of 0 dB. Use the cursors (Cursor button) to measure frequencies (Type - Frequency) or amplitudes (Type - Amplitude). Increase span when needed. The effective value of this signal will be the reference voltage. To measure the effective value after adjusting the fundamental at 0 dB, proceed as follows: press the Autoset button, then press the Measure button, select one of the 5 simultaneous measurements available by pressing a button from the 5 ones situated on the right of the screen. Select the source channel to which the generated sinusoidal signal was connected, and Cyc RMS as a measurement type.

I) The spectra of theoretical and experimental amplitudes for the studied signals are plotted on a millimetre paper, $\left.\frac{A_k}{A_1}\right|_{\text{teoretic}}$ and $\left.\frac{A_k}{A_1}\right|_{\text{experimental}}$, depending on the frequency.

For the same duty cycle, the spectra are plotted on the same graph (the theoretical value through a segment, and the experimental value by one point, using a different colour for segments and another for points).

For the triangular signal, the theoretical and experimental amplitude spectra are plotted on another millimetre paper, $\left.\frac{A_k}{A_1}\right|_{\text{teoretic}}$ and $\left.\frac{A_k}{A_1}\right|_{\text{experimental}}$, on a frequency domain, on the same coordinate axes.

J) The power dissipated on a 1Ω resistor is determined for the rectangular signals based on the amplitude spectrum (table 2) using equation (12)

Compare the power obtained using the experimental data P_e and the power of the fundamental P_1, with the power calculated based on the time expression, P_t, equation (13). The $\frac{P_e}{P_t}$ and $\frac{P_1}{P_t}$ ratios are determined.

K) The power of the triangular signal based on the amplitude spectrum is determinate, relation (6)

The P_e is compared with P_t power which is calculated using the equation (7). The $\frac{P_e}{P_t}$ and $\frac{P_1}{P_t}$ ratios are determined, where P_1 is the power of the fundamental component.

1.4. Preparatory Questions

a) Determine $A_1[\mathrm{V}]$ and $A_2[\mathrm{dBm}]$ using $A_1 = 20\ \mathrm{dBm}$ and $A_2 = 0{,}01A_1$, $U_{ref} = 0{,}2236\ \mathrm{V}$. Repeat for $U_{ref} = 0{,}775\ \mathrm{V}$.

b) If $A_1 = 20\ \mathrm{dBm}$ and $A_2 = 0{,}01A_1$, which is the difference (in dB) between $A_1[\mathrm{dBm}]$ and $A_2[\mathrm{dBm}]$? Repeat for $A_2 = 0{,}1A_1$ and $A_2 = 0{,}001A_1$. What do you observe?

c) If $A_1 = 20\ \mathrm{dBm}$ and $A_2 = 14\ \mathrm{dBm}$, which is the value of the $\frac{A_2}{A_1}$ ratio in level units. Specify the unit of measurement.

d) Determine the power dissipated by a harmonic signal with a level of $0\,\mathrm{dBm}$ ($U_{ref} = 0{,}2236\ \mathrm{V}$) on a resistor of $50\,\Omega$, $75\,\Omega$, $600\,\Omega$. Repeat for a level signal of $10\,\mathrm{dBm}$. What power increase determines the change with $10\,\mathrm{dB}$ the signal level?

e) Determine the power dissipated by a harmonic signal with a level of $0\,\mathrm{dBm}$ ($U_{ref} = 0{,}775\ \mathrm{V}$) on a resistor of $50\,\Omega$, $75\,\Omega$, $600\,\Omega$. Repeat for a level signal of $10\,\mathrm{dBm}$. What power increase determines the change with $10\,\mathrm{dB}$ the signal level?

f) Draw the amplitude and phase spectra for the signal $s(t)=2+2\sin(100t+\pi)-3\cos(200t)+\cos^2\left(400t-\frac{\pi}{4}\right)$.

g) A periodic signal was measured with a spectrum analyzer. The following values were obtained: $A_1=20\ \text{dBm}$, $A_2=10\ \text{dBm}$, $A_3=-25\ \text{dBm}$, $A_4=1\ \text{dBm}$, $A_5=-21\ \text{dBm}$, $A_6=-25\ \text{dBm}$ and $A_7=-30\ \text{dBm}$. Determine the effective bandwidth of the signal if the limit is $0,01A_1$, $0,1A_1$, respectively $0,001A_1$.

1.5. Questions

a) What is the value of the DC offset for the studied signal in B and C?

b) What is the rise time for an ideal rectangular signal?

c) Why we cannot obtain a perfect extinction (suppression) of the even harmonics when $\tau/T=1/2$?

d) Two rectangular periodic signals have the same period and complementary duty factors $\tau_1/T+\tau_2/T=1$. What is the relationship between the amplitudes A_k of the two signals?

1.6. Exercises

a) Adjust the parameters of an periodic rectangular signal such that $T=50\ \mu\text{s}$, $\tau/T=1/3$, $A=U_r$. Determine the values of the amplitudes A_k, $k=0$.

b) When a sine signal was measured, the level of the harmonics are the following: $n_1=-3\,\text{dB}$, $n_2=-43\,\text{dB}$, $n_3=-49\,\text{dB}$, $n_4=-63\,\text{dB}$ ($U_{ref}=1\ \text{V}$). Determine the harmonic of 1st order (in mV) and the distortion factor.

c) In the spectral analysis of a periodic rectangular signal it was found that the 2nd order harmonic has with 25 dB less than the fundamental. What duty cycle does the analyzed signal have? What will be the difference in dB between the 1st order harmonic and the 3rd harmonic level for this signal?

d) The signal in a) is applied at the input of an ideal low pass filter with the cut-off frequency $f_t=45\,\text{kHz}$. Make a graphical representation of the output signal

ANNEXES

Instructions for use for the devices

The Function Generator GFG 301

1) Setting the frequency: press FREQ button, introduce the desired frequency value and then choose the measurement unit by pressing the appropriate button (i.e., “kHz/Vrms”).

2) Setting the type of the function: press FUNC button repeatedly until in the up left side of the function generator screen is lit the shape corresponding to the desired function, which will be automatically generated (triangular/harmonic/rectangular signal).

3) Setting the amplitude E: press AMPL button, introduce the desired value and then choose the measurement unit by pressing the appropriate button (i.e., “Hz/Vpp”). The keys ◀,▶ can be used to change the input value. The rotary knob can be used to increase or decrease that input value, such that it can be obtained A_1=0 dBm on the spectrum analyzer.

4) Adjusting the duty cycle: press “DUTY” button, introduce the desired value and then choose the measurement unit by pressing the appropriate button (i.e., “DEG/%”).

The spectrum analyzer GSP810

1) Setting the centre frequency (the working frequency): press CENTER button, introduce the desired centre frequency value in MHz and then press ENTER the measurement unit by pressing the appropriate button.

2) Setting the frequency on a division (SPAN): press SPAN, use the rotary knob to set the desired value.

3) Setting the resolution of the frequency band: it is automatically adjusted when the value for SPAN is set.

4) Markers/Cursors: press MKR button to enable the markers/cursors on the display. The first marker/cursor is automatically selected. With the help of the arrow keys near the rotary knob, together with the rotary knob will modify the frequency values. Switching from one marker/cursor to another is made with the

ENTER key. The attenuation (in dBm) corresponding to the frequency at which the cursor is fixed is displayed next to the marker.

Digital Oscilloscope TDS 1001

1) To view the signal $x(t)$ on channel 1 (CH 1), proceed as follows: connect the signal to the BNC jack corresponding to channel 1 and press the CH1 key. From the **VOLTS/DIV** (amplitude) knob, match the image of the signal such that it occupies as much as possible from the screen of the oscilloscope. The larger the image is on the screen, the more accurate is the reading.

2) The vertical positioning (the vertical movement) of the signal can be done with „POSITION⇳" button.

3) The horizontal positioning (the horizontal movement) of the signal can be done „POSITION⬄" button.

4) The adjustment SEC/DIV button (time base period) modifies the number of periods of the $x(t)$ signal viewed on the screen. For a correct visualization, a 1½ period of the signal is framed on the oscilloscope screen.

5) To measure and/or to compare amplitudes, 2 cursors can be used, which are enabled by pressing „CURSOR" button. On the Ox axis are enabled „Type Voltage" and „Source CH1" buttons located on the top right side of the oscilloscope screen (the first 2 buttons). The movement of the cursors is made with the „POSITION⇳" button. The values associated the 2 cursors are read from the right side of the screen.

6) To view the signal $x(t)$ in the frequency domain, proceed as follows: press the „MATH MENU" button. From the „SEC/DIV" button, the signal is positioned on the frequency axis. To enable the cursors, press the „CURSOR" button. For the amplitude axis enable the 2 buttons from the top right-side screen of the oscilloscope, „Type Amplitude" and „Source MATH". The movement of the cursors is made with the „POSITION⇳" button. The values associated the 2 cursors are read from the right side of the screen.