Laboratory 1

PERIODIC SIGNALS

1.1. The objective of the laboratory

The paper focuses on the spectral analysis of periodic signals. In order to achieve this, we have to measure the amplitude spectrum of sinusoidal and rectangular periodic signals, with different duty cycles, as well as the amplitude spectrum of triangular symmetrical periodic signals. We have to determine the power obtained for the square signal (with different duty cycles) and the one for the triangular signal, using the experimental data. Lastly, the resulting power will be compared to the one obtained through the time domain representation of the same signals.

1.2. Theoretical aspects

A periodic signal $x(t)$ is mathematically represented as a periodic function, in other words, for which there is a real nonzero number T, called *period*, such that the following equality is achieved:

$$x(t+T)=x(t), \quad \forall t \in \mathbb{R}. \tag{1}$$

If T is period and ensures the fulfilment of equation (1), then any multiple, kT, where $k \in \mathbb{Z}^*$, is also a period for the signal. The lowest, strictly positive value of the period is called the *main period* (or *repetition period*) of the signal.

The usual signals encountered in practice have a moment of occurrence and a moment of extinction, in other words they can fulfil equation (1) only on a finite length of the time axis, which means that rigorous periodic signals do not exist in practice. However, in some situations, it is useful to model a finite-length signal, having on its existence a periodic variation, using a periodic function that fulfils (1) on all the real axes. This modelling does not lead to errors if the duration of the signal is much higher than the repetition period and than the duration of the transient regimes occurring in the circuit when applying or suppressing the signal and, moreover, if these transient regimes are not in the area of interest.

A periodic signal $x(t)$, by T period, can be developed in Fourier Series if the Dirichlet conditions are satisfied.

The formulas of the Fourier Series and the relations used for calculation of the coefficients are given in Table 1.

Table 1 The Fourier series expressions of an analogic periodic signal

THE SERIES TYPE	**ANALYTIC REPRESENTATION**	**RELATIONS FOR THE COEFFICIENTS**
Exponential (complex)	$x(t)=\sum_{k=-\infty}^{\infty}A_{kc}e^{jk\omega_0 t}$	$A_{kc}=\frac{1}{T}\int_{t_0}^{t_0+T}x(t)e^{-jk\omega_0 t}dt$
Trigonometric	$x(t)=c_0+\sum_{k=1}^{\infty}c_k\cos k\omega_0 t+$ $+\sum_{k=1}^{\infty}s_k\sin k\omega_0 t$	$c_0=\frac{1}{T}\int_{t_0}^{t_0+T}x(t)dt$ $c_k=\frac{2}{T}\int_{t_0}^{t_0+T}x(t)\cos k\omega_0 t dt$ $s_k=\frac{2}{T}\int_{t_0}^{t_0+T}x(t)\sin k\omega_0 t dt$
Cosine (Harmonic)	$x(t)=A_0+\sum_{k=1}^{\infty}A_k\cos(k\omega_0 t+\varphi_k)$	$A_0=c_0$ $A_k=\sqrt{c_k^{\ 2}+s_k^{\ 2}}=2\lvert A_{kc}\rvert$ $\varphi_k=-arctg\frac{s_k}{c_k}=\arg\{A_{kc}\}$

$\omega_0=\frac{2\pi}{T}=2\pi f_0$ represents the *fundamental angular frequency* (pulsation), and f_0 is the *fundamental frequency*, which is also called the *repetition frequency* of the periodic signal.

The choice regarding the limits of integration while evaluating the coefficients from the Fourier Series is arbitrary, it is made in such a way as to simplify the calculations; it is very important that the integration is made over a period (from $-T/2$ to $+T/2$, from 0 to T, etc.).

The Exponential Fourier Series provides a decomposition of the periodic signal into a sum of elementary exponential components $e^{jk\omega_0 t}$, physically unachievable. Its use is very convenient in the problems in which is required to determine the response of circuits to periodic signals.

Practically (experimentally), we are interested in the Harmonic Fourier Series (HFS). The following details are only applicable to this expansion. This decomposes the signal into a sum of cosine signals (hereinafter referred to as *components*), the frequencies of which are equal to multiples of the repetition frequency of the periodic signal. These components are also known as *harmonics*. The *component* situated on the zero frequency is called *DC offset* (*the continuous component*), the component at the f_0 frequency is *the fundamental component* (often called "the *fundamental*", *"the first order harmonic"* or *"the repetition frequency"*), and the components situated at the frequencies $kf_0\left(k \in \mathbb{N}, k \geq 2\right)$ are *the harmonic components* ("the *harmonics of* k *order*"). The assembly of these components forms the *spectrum of the signal*. Note that, in the case of periodic signals, the *spectrum is discrete*, with components only at certain frequencies, since periodic signals can be represented by *discrete sums* of elemental signals, as shown in Figure 1.

The characterization of the periodic signals, in the frequency domain, is made using *the amplitude spectrum* and *the phase spectrum*, meaning a graphical representation of the amplitude-frequency dependencies and, respectively, of the initial phase-frequency dependencies of the components. For this purpose, to each component from the development is assigned a straight segment (*spectral line*) in the two spectra, *located at the frequency of the component* and having the amplitude of the segment proportional to the amplitude or phase of the component.

The sign "x" in the amplitude spectrum shows that the amplitudes are null and in the phase spectrum it shows that, in those cases, the notion of the initial phase is meaningless or not determined; the component at $f = 4f_0$ does not exist (has zero amplitude, so there is no need to determine the initial phase of a zero amplitude signal). The case of periodic signals with DC offset is not discussed in this paper.

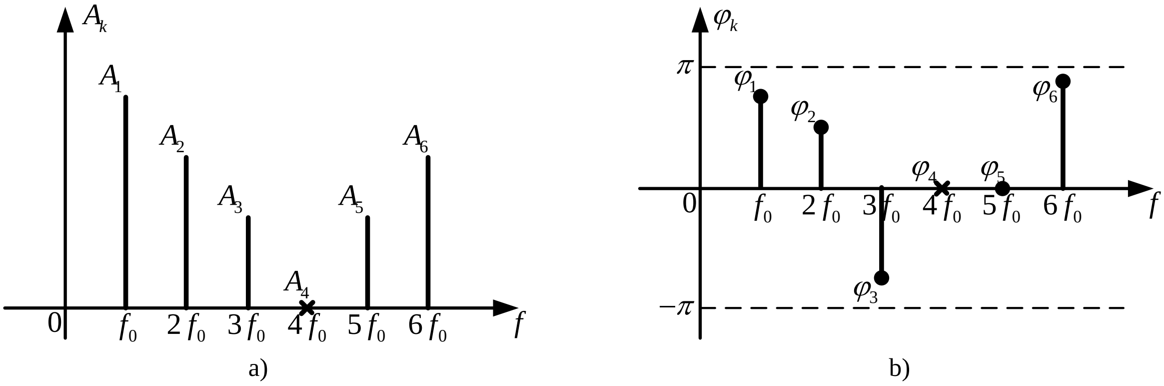


Figure 1. a) Amplitude spectrum diagram; b) Phase spectrum diagram

It is noted that, it is sufficient to know the amplitude and phase spectra to completely determine the signal.

Theoretically, the signal spectrum ranges from zero frequency, $f = 0$, to the infinite frequency, $f = +\infty$; in practice, the components situated at very high frequencies are negligible because of their small amplitude values, so for signals used in practice, the bandwidth of the signal is finite, meaning, the spectrum is limited. The decrease in amplitude of the components when the frequency increases is even faster as the signal is smoother (the mathematical function used for the representation is derivable as many times as possible). The spectral truncation depends on the requirements imposed by the type of communication which is using the signal. Therefore, the spectral analysis of a signal allows us to determine the *effective bandwidth occupied* by that signal.

It is called " *effective bandwidth*" the bandwidth occupied by important components of the considered application. The effective bandwidth depends on the threshold value below which the components from the amplitude spectrum can be considered as negligible. When the threshold value increases, the effective bandwidth decreases; the choice of the negligence threshold is made according to a criterion established on practical considerations for each application. If the effective bandwidth of the signal is known, it is possible to determine the bandwidth in which the circuits that process the signal function correctly.

A. The harmonic signal

The analytical expression of a harmonic signal is:

$$x(t) = A \cdot \cos(\omega_0 t + \varphi), \tag{2}$$

and the graphical representation of the amplitude spectrum is shown in Figure 2, where $f_0 = \frac{1}{T} = \frac{\omega_0}{2\pi}$.

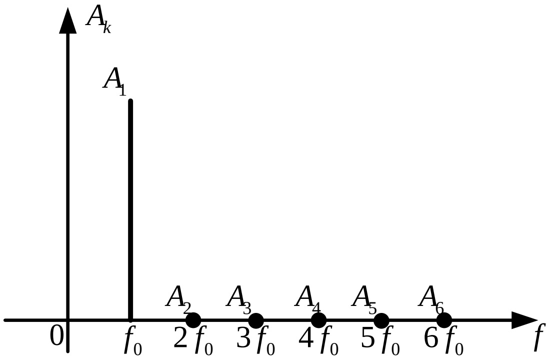


Figure 2. The diagram of the amplitude spectrum of a harmonic signal

A pure harmonic signal has a distortion factor of 0% (does not have harmonics with an order higher than 1). The real signal obtained from the function generator used in the laboratory is not perfectly sinusoidal, which implies the presence of non-zero spectral components for frequencies that are multiples of the fundamental frequency. We are interested in finding out how much the signal from the function generator differs from the pure harmonic signal or, in other words, how distorted (modified) the generated harmonic signal is; the distortions appear due to inherent nonlinearities in the generator's circuits. To measure these distortions the harmonic distortion factor was introduced δ, and it is defined as follows:

$$\delta = \frac{\sqrt{A_2^2 + A_3^2 + \ldots\ldots}}{A_1} = \sqrt{\sum_k 10^{\frac{n_k - n_1}{10}}}, \tag{3}$$

where, $n_k = 20\lg\frac{A_k}{U_r}$, and the reference voltage, U_r, will be explained later.

δ must be as small as possible, close to zero.

B. The triangular signal

Using Table 1, the Harmonic Fourier Series of the symmetrical triangular periodic signal is calculated, with repetition frequency f_0 (Figure 3.a):

$$x(t)=\sum_{k=1}^{\infty}2E\,\mathrm{sinc}^2\frac{k\pi}{2}\cos k\omega_0 t=\sum_{n=0}^{\infty}\frac{8E}{\pi^2(2n+1)^2}\cos(2n+1)\omega_0 t\,. \qquad (4)$$

From (4), the amplitude of the spectral components is identified:

$$A_k=\begin{cases}\dfrac{8E}{\pi^2k^2} & ,k\text{ odd}\\ 0 & ,k\text{ even}\end{cases}\,. \qquad (5)$$

The graphical representation of the amplitude spectrum of the signal from Figure 3.a) is given in Figure 3.b), and the amplitude spectrum normalized to the amplitude of the fundamental component is found in Figure 3.c).

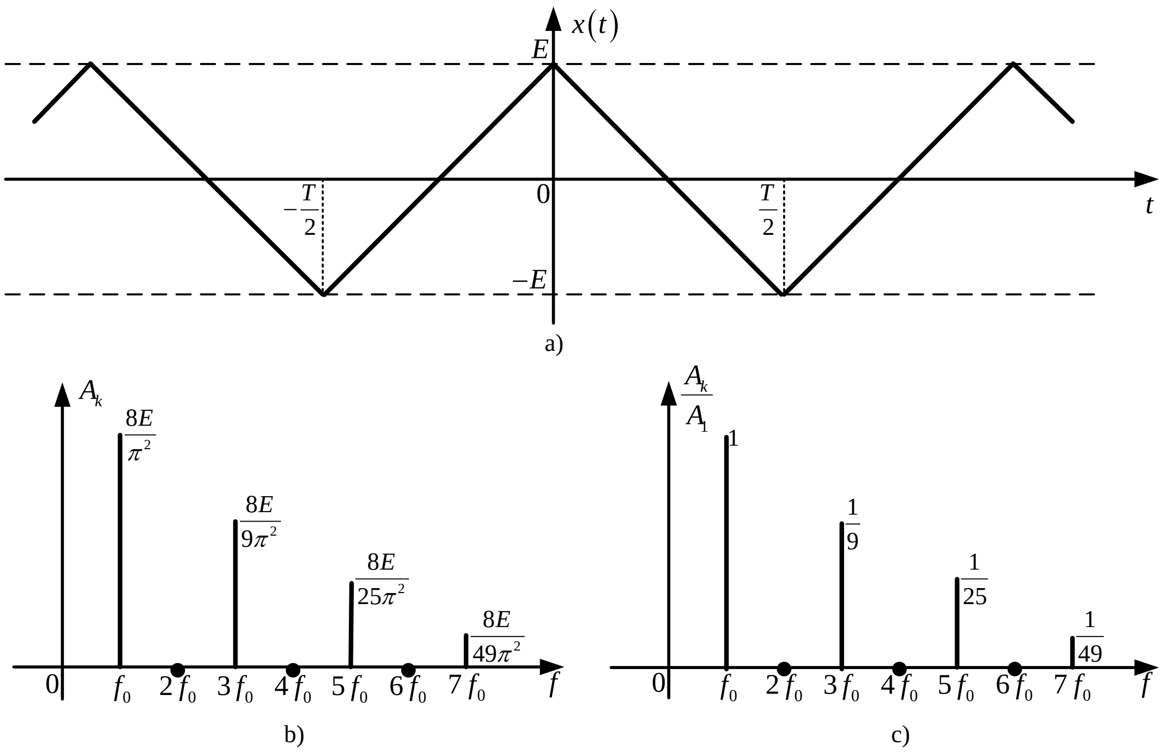


Figure 3. a) Time domain representation of the triangular signal; b) Amplitude spectrum for the triangular signal; c) Normalized amplitude spectrum for the triangular signal

The power of the symmetrical triangular signal, dissipated on a 1Ω resistor, can be calculated based on experimental data according to the relationship:

$$P_e = \sum_{k=1}^{k_M} \frac{A_k^2}{2}, \tag{6}$$

where k_M is the number of harmonics that are found in the amplitude spectrum.

If the time domain representation of the signal is used, the power of the triangular signal on a 1Ω resistor, is calculated as follows:

$$P_t = \frac{1}{T}\int_{(T)} |x(t)|^2\, dt = \frac{E^2}{3} = X_{ef}^2, \tag{7}$$

where X_{ef} is the effective value of the analysed signal.

C. The rectangular signal

The graphical representation of a rectangular signal is shown in Figure 4. The signal is even, so the amplitudes s_k, of the trigonometric series are null and $A_k = |c_k|$.

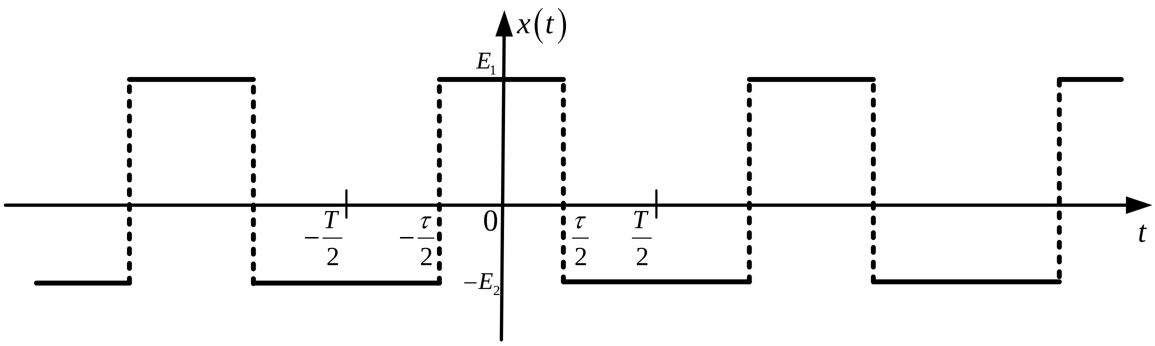


Figure 4. The graphic representation of the rectangular signal without DC offset and duty cycle $\frac{\tau}{T}$

From the point of view of the amplitude spectrum, the signal parity does not matter because the movement along the time axis only results in the modification of the phase spectrum, φ_k, the amplitude spectrum, A_k, will be the same.

Using the relationships from Table 1 the expression of the Harmonic Fourier Series is:

$$x(t)=\sum_{k=1}^{\infty}\frac{2(E_1+E_2)}{k\pi}\sin(k\pi\frac{\tau}{T})\cos(k\omega_0 t). \qquad (8)$$

From (8) the expression of A_k becomes:

$$A_k=2(E_1+E_2)\frac{\tau}{T}\left|\frac{\sin(k\pi\frac{\tau}{T})}{k\pi\frac{\tau}{T}}\right|=2(E_1+E_2)\frac{\tau}{T}\left|\mathrm{sinc}(k\pi\frac{\tau}{T})\right|, \qquad (9)$$

which highlights the fact that the amplitudes of the signal harmonics decrease after a $|\mathrm{sinc}x|=\left|\frac{\sin x}{x}\right|$ - shaped envelope. It is important to notice their proportionality with the amplitude E_1+E_2 of the periodic rectangular signal, as well as with the ratio τ/T, called *duty cycle*.

When the rectangular signal has a duty cycle $\frac{\tau}{T}=\frac{1}{2}$, the amplitudes E_1 and E_2 are equal in module ($|E_1|=|E_2|$). If the duty cycle decreases, $|E_1|$ increases and $|E_2|$ decreases to obtain a zero DC offset. Changing the duty cycle without changing the amplitudes $|E_1|$ and $|E_2|$ leads to changing the DC offset.

The harmonics for which the following condition is met $k\pi\frac{\tau}{T}=p\pi$ (meaning $k=p\frac{T}{\tau}$), p being an integer, have zero amplitudes. For example, for $\frac{\tau}{T}=\frac{1}{2}$ and $\frac{\tau}{T}=\frac{1}{10}$, the even harmonics will be null, $k=2p$, respectively harmonics of $k=10p$ order.

Figure 5 shows the amplitude spectrum of the signal $x(t)$ from Figure 4, maintaining the T period constant, and the τ width of the impulse equal to $T/2$, respectively $T/10$.

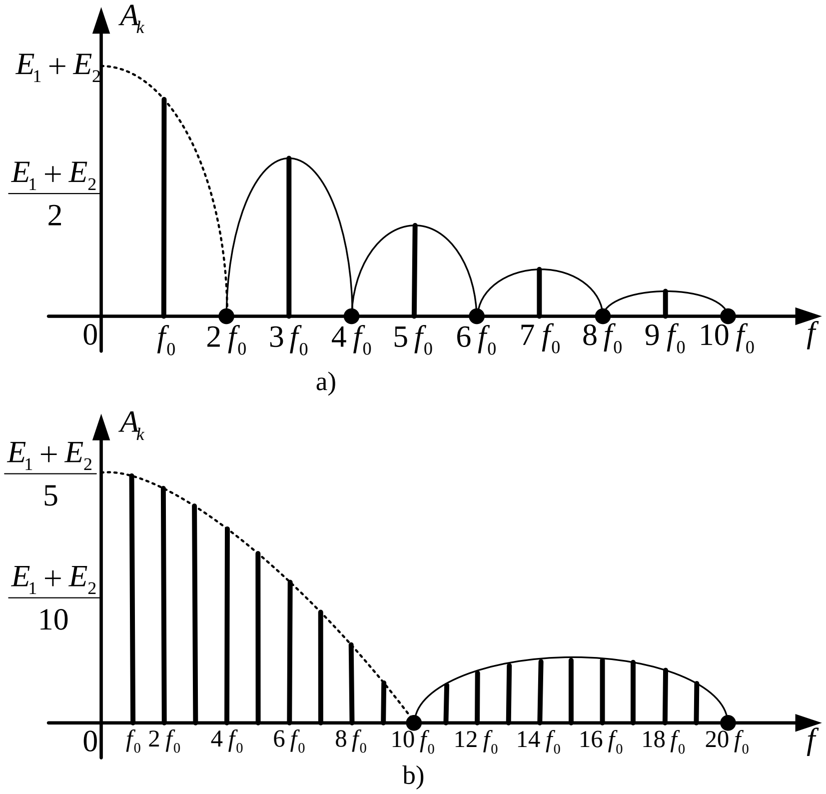


Figure 5. a) Amplitude spectrum of the rectangular signal with duty cycle $\tau/T = 1/2$

b) Amplitude spectrum of the rectangular signal with duty cycle $\tau/T = 1/10$

The normalized amplitude spectra are obtained by the normalization of A_k at the fundamental amplitude value A_1.

$$\left|\frac{A_k}{A_1}\right| = \frac{1}{k} \cdot \frac{\left|\sin k\pi \frac{\tau}{T}\right|}{\sin \pi \frac{\tau}{T}}. \tag{10}$$

This ratio highlights a decrease in amplitude of the harmonics compared to the fundamental.

So, for a $\tau/T = 1/2$ the relationship (10) becomes:

$$\left|\frac{A_k}{A_1}\right| = \frac{1}{k} \cdot \frac{\left|\sin \frac{k\pi}{2}\right|}{\sin \frac{\pi}{2}} = \begin{cases} \frac{1}{k} & , k \text{ odd} \\ 0 & , k \text{ even} \end{cases} . \tag{11}$$

The graphical representation of the normalized amplitude spectrum, in this case, is given in Figure 6.

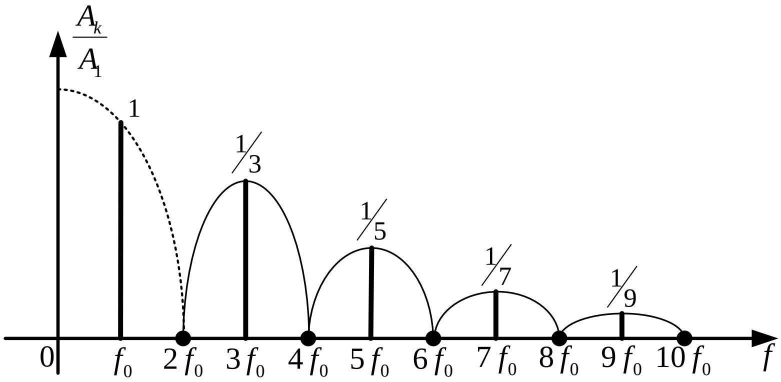


Figure 6. Graphical representation of the amplitude spectrum normalized to the fundamental frequency for a rectangular periodic signal with duty cycle $\tau/T = 1/2$

The power of the rectangular signal, dissipated on a 1Ω resistor, can be calculated based on experimental data according to the relationship:

$$P_e = \sum_{k=1}^{k_M} \frac{A_k^2}{2}. \tag{12}$$

If the time domain representation of the signal (Figure 4) is used, the power of the rectangular signal strength on a 1Ω, is calculated using the relation:

$$P_t = \frac{1}{T}\int_{(T)} \left|x(t)\right|^2 dt = \frac{\tau}{T}\left(E_1^2 - E_2^2\right) + E_2^2. \tag{13}$$

1.3. The experimental part of the laboratory

The block diagram of the assembly is shown in Figure 7.

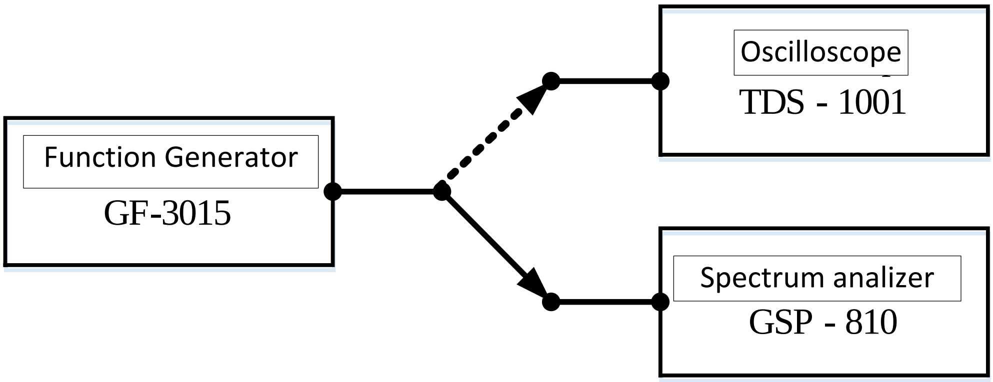


Figure 7. The block diagram of the assembly for periodic signals

A) Determination of working parameters for the spectrum analyzer

When making measurements, reading them and interpreting the measured values, take into account that V_{rms} is the effective value of the voltage expressed in volts (rms = root mean square). It helps in identifying various voltage parameters of a signal (average voltage, peak voltage, amplitude etc – consult the METc lecture/laboratory).

For the devices used in the laboratory, dBm is the unit of measure for the voltage expressed in decibels having the reference voltage the voltage which corresponds to a power of 1mW on a resistance of 50 Ω. So, for a $P = 1\,mW$ and a $R = 50\,\Omega$, the actual reference voltage is obtained as follows:

$$U_{r,ef} = \frac{U_r}{\sqrt{2}} = \sqrt{PR} = \sqrt{10^{-3} \cdot 50} = 0,2236\ V\ .$$

This voltage is used when there exists a perfect matching between the generator and spectrum analyzer, which is equivalent to saying that the output impedance of the generator and the input impedance of the spectrum analyzer (assuming they are complex) must be conjugate complex ($Z_{OUTgenerator} = Z^{*}_{IN\ analyzer}$). The two impedances form a voltage divider (figure 8). Generally, this condition is fulfilled, hence the reference voltage is (generally!) 0.2236 V_{rms}.

This step of the practical work aims to initiate the students in their using of a measuring bench, which they have never worked with before. In practice, the first operation is checking whether the devices work properly. The output impedance of the signal generators is value of 50Ω. Therefore, the value of the input impedance of the analyzer must be determined. The effect of its deviation at the value of 50Ω can be expressed through a corresponding deviation of the reference voltage, with the value of 0.2236 V_{rms}. This is rather convenient, especially for the calculations that are to be made in this paper.

It is worth noting that the apparatus always transforms in dBm using the reference voltage 0.2236 V_{rms}, because this is a parameter only used for the calculations made by the apparatus. Since the devices have identical processors, they clearly cannot differ from one device to another. Hence, in practice, the parameter that can differ in the case of generators used in this practical work is the input impedance, which can be the equivalent of using a different reference voltage for the calculations (hereinafter named *equivalent reference voltage*).

To determine the equivalent reference voltage, the further steps are followed:

- A sinusoidal signal is applied at the input of the oscilloscope, with the frequency of 200 kHz, obtained using the function generator. For the connections, consult figure 7 (Press the Waveforms button and the corresponding button underneath the screen for a harmonic wave form). For setting the frequency, press the button underneath the screen corresponding to the Frequency parameter, until this parameter is highlighted in blue. Then, introduce from the keyboard the value of 200 and from the buttons underneath the screen, press the corresponding one for the unit of kHz;
- Measure the effective value of the generated signal with the oscilloscope as follows: activate the channel of the oscilloscope which is connected to the function generator by pressing the 1 or 2 button (it must turn yellow). Press the Auto Setup button, then the Measure one, select the channel to which the generated sinusoidal signal was connected to be the source (by pressing the button corresponding to Source underneath the oscilloscope's screen, until the desired channel is selected, then press the rotating button Intensity/Adjust to save the settings). Next, activate the All Measure (by pressing the button underneath the oscilloscope's screen corresponding to All Measure, until it is On). Look at the value given by RMS. Adjust the generated signal's amplitude such that the effective value V_{rms} indicated on the oscilloscope to be $2 \cdot U_{r,ef} = 0,4472$ V_{rms} (by

pressing the button underneath the function generator's screen corresponding to Amplitude such that Amplitude is highlighted in blue. Using the rotating button of the function generator, adjust the amplitude until RMS displays 0.4472). The value displayed on the signal generator will be U_1.

- Disconnect the cable from the oscilloscope and connect it to the spectrum analyzer. Measure using the cursor the voltage U_2 in dBm, U_2 [dBm] (figure 8) as follows: press the CENTER button, input 0.2 and press the MHz button. Press SPAN and adjust this parameter's value using the rotating button at 10 kHz/div. Press the MKR button, input 0.2 and press MHz. Read the displayed value in dBm, pointed by the cursor placed at 0.2 MHz. Be careful, the device always indicates the sign (+ or -), which is important. Such an example is: **1:0.200 – 3.4 dBm.**

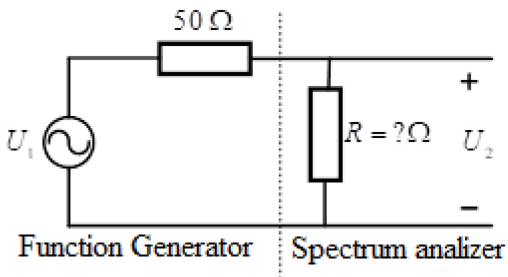
- To calculate the voltage divider, relation (15), transform U_2 in volts using the formula $U_2[\mathrm{V}] = 0,2236 \cdot 10^{\frac{U_2[\mathrm{dBm}]}{20}}$

$$\frac{U_2}{U_1} = \frac{R}{50+R} = \theta,$$

where θ is the division coefficient.

- From relation (15) determine the value of R and

$$U_{r,ef,real} = \sqrt{10^{-3} \cdot R}\ \mathrm{V}$$



As a reminder, for a voltage U, its value in dBm is:

$$n = 20\lg\frac{U}{U_r} \quad [\mathrm{dB}],$$

where U_r is a reference voltage.

B) Theoretical and experimental spectral analysis of the rectangular periodic signal with duty cycle $\tau/T = 1/2$.

Set the function generator's waveform (Waveforms button) to be rectangular, the frequency (Frequency button) $f_0 = 200\,\text{kHz}$, the duty cycle (DUTY button) $\tau/T = 1/2$ (i.e. 50%) and the amplitude (Amplitude button) E of the rectangular signal such that the level of the fundamental harmonic (the spectral component placed at the signal frequency, in this case $200\,\text{kHz}$) measured with the spectrum analyzer to be $0\ \text{dBm}$. To avoid setting errors, set the reference level to $10\ \text{dBm}$ - REF LVL button. To measure the harmonics, the centre frequency of the analyzer ("CENTER" button) is fixed at $1\,\text{MHz}$ and SPAN at $200\,\text{kHz/div}$ (in this way you can see more harmonics on the analyzer screen, starting with the fundamental harmonic). One of the markers is adjusted to the harmonic frequency of the component which is to be measured and read the value indicated in dBm (for example, it is desired to measure the second harmonic. The cursor is adjusted to $0{,}400\ \text{MHz}$ and read the value indicated in dBm. Because the second harmonic is measured, the measured value will represent A_2 expressed in dBm. These notations are used in the calculations below. When the cursor indicates HIGH or LOW it means that it has been set a higher or a lower value of the maximum frequency, respectively, of the minimum frequency that can be viewed on the analyzer screen, which means that the analyzer centre frequency must be changed to a higher value, respectively smaller value.

Measure the level in dBm of the first 20 harmonics (A_1, A_2 etc.) and write them down separately in Table 2. In Table 2 we have:

- k - the order of harmonics,
- $f_k\left[\text{MHz}\right]$ - the harmonic frequency of k order,
- For the experimental part:

  - Using markers A_1, A_2, etc. were measured in dBm;

  - Write in the table $\left.\dfrac{A_k}{A_1}\right|_{\text{exp}}\left[\text{dB}\right] = A_k\left[\text{dBm}\right] - A_1\left[\text{dBm}\right]$.

For the theoretical part (calculated at home):

- $\left.\dfrac{A_k}{A_1}\right|_{\text{th.}}$ - is calculated with equation (11);

- $\left.\frac{A_k}{A_1}\right|_{\text{th.}} [\text{dB}] = 20\lg\frac{A_k}{A_1}$.

Table 2 Spectral analysis of the rectangular periodic signal with $\tau/T = 1/2$

k	1	2	3	4	5	...	19	20
$f_k\,[\text{MHz}]$	0,2	0,4	0,6	0,8	1	...	3,8	4
$\left.\frac{A_k}{A_1}\right\|_{\text{th}}$								
$\left.\frac{A_k}{A_1}\right\|_{\text{th}} [\text{dB}]$								
$\left.\frac{A_k}{A_1}\right\|_{\text{expe}} [\text{dB}]$								
$\left.\frac{A_k}{A_1}\right\|_{\text{expe}}$								

The values E_{01} and $-E_{02}$ of the studied signal are measured using the oscilloscope: press the Auto Setup button, then press the Measure button so that it is on (to select a value, press the rotating button Intensity/Adjust). Select as the source the channel which the studied signal was connected to (by pressing the button underneath the screen corresponding to Source to select the desired channel) and read the values Max (for E_{01}), respectively Min (for E_{02}), from the screen.

C) Theoretical and experimental spectral analysis of the rectangular periodic signal with duty $\tau/T = 1/4$.

The theoretical and experimental spectral analysis of the same rectangular periodic signal with $f_0 = 200\,\text{kHz}$, but with $\tau/T = 1/4$ (i.e., 25%, change made by pressing the DUTY button and adjusted by using the rotating button). Pay attention to the E amplitude (Amplitude button), after changing the duty, the amplitude of E is set such that the level of the fundamental harmonic is equal to 0 dBm, similar to the experiment from point B. The calculations are like those from the previous point.

Table 3 Spectral analysis of the rectangular periodic signal with $\tau/T = 1/4$

k	1	2	3	4	5	...	19	20
f_k [MHz]	0,2	0,4	0,6	0,8	1	...	3,8	4
$\left.\frac{A_k}{A_1}\right\|_{th}$								
$\left.\frac{A_k}{A_1}\right\|_{th}$ [dB]								
$\left.\frac{A_k}{A_1}\right\|_{expe}$ [dB]								
$\left.\frac{A_k}{A_1}\right\|_{expe}$								

The values E_{01} and $-E_{02}$ of the studied signal are measured as explained at the previous point.

D) Measuring the rise time for the periodic rectangular signals studied above: t_{c1}($\tau/T = 1/2$) and t_{c1} ($\tau/T = 1/4$).

Connect the function generator to the oscilloscope. Adjust the deflexion coefficient on the ox axis (horizontal) (sec/div) (the rotating button from section Horizontal) to the minimal value, by pressing the button All Measure underneath the screen. Select the channel connected to the studied signal as the source, then read the Rise time displayed on the screen.

E) Determine the bandwidths occupied by the rectangular periodic signals studied above.

It is taken into account that in the bandwidth are included all the components that have amplitudes higher than 1% of the fundamental amplitude, meaning $0{,}01 \cdot A_1[\mathrm{V}]$. Firstly, the amplitude of the fundamental is set at 0 dBm, like in the above experiments. That is, $20\lg\frac{A_1}{U_r} = 0\,[\mathrm{dBm}]$. So, $0{,}01 \cdot A_1[\mathrm{V}]$ expressed in dBm will be:

$$20\lg\frac{0,01A_1}{U_r} = 20\lg 0,01 + 20\lg\frac{A_1}{U_r} = -40 + 0 = -40\,[\text{dBm}].$$

Therefore, we will be looking for all the components that have amplitudes higher than $-40\,\text{dBm}$. Taking in consideration that in the case of a rectangular signal, with duty cycle 50%, the harmonics of even order are very small, practically zero, we will be out of the bandwidth only when minimum 3 consecutive harmonics have the level less than $-40\,\text{dBm}$. In the case of a rectangular signal, with duty cycle 25%, the harmonics of $k = 4p,\ p \in \mathrm{N}^*$ order are very small (practically zero), so we are out of the bandwidth when minimum 5 consecutive harmonics have the level less than $-40\,\text{dBm}$. For performing experiments only at the limit of the bandwidth we proceed as follows: at the function generator it is set one of the above periodic signals. Pay attention to the correct adjustment of the E (Amplitude button) after changing the duty cycle (DUTY button) such that the level of the fundamental is $0\,\text{dBm}$, similar to the experiment at point B. Adjust the reference level of the spectral analyzer to $10\,\text{dBm}$ (REF button LVL), that is the maximum measurable level. If all the adjustments were made correctly, the fundamental harmonic level is near a horizontal black line drawn from the spectrum analyzer screen (see Figure 9), line indicating a level equal to $0\,\text{dBm}$. A vertical division of the screen has $10\,\text{dBm}$, so to identify the horizontal line of $-40\,\text{dBm}$ we have to identify the fourth horizontal black line below that of the $0\,\text{dBm}$ horizontal line. Then, it is increased the value indicated by CENTER until the harmonics are close to $-40\,\text{dBm}$ (see Figure 10).

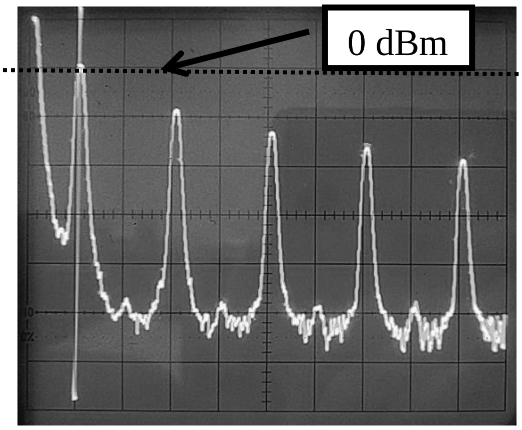


Figure 9. The spectrum of the rectangular signal with duty cycle 50% (low frequencies).

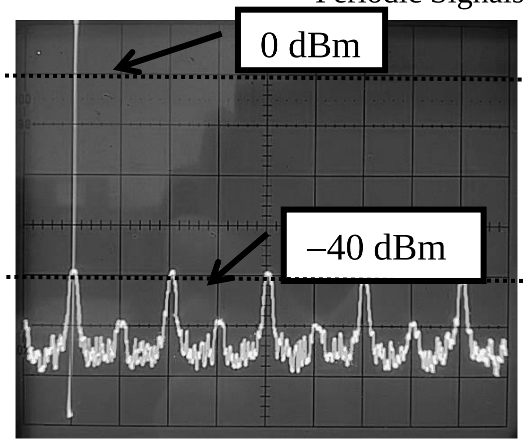


Figure 10. The spectrum of the rectangular signal with duty cycle 50% (high frequencies).

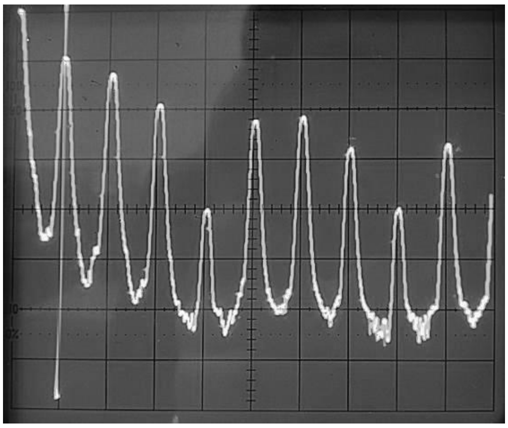


Figure 11. The spectrum of the rectangular signal with duty cycle 25% (low frequencies).

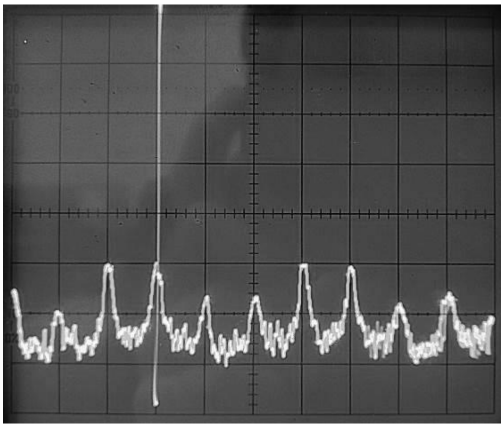


Figure 12. The spectrum of the rectangular signal with duty cycle 50% (high frequencies).

Place one of the cursors on the screen, set the frequency value equal to the value indicated by CENTER. Considering that the generator is set to a signal of $200\,\text{kHz}$, its harmonics are located on multiple of $200\,\text{kHz}$. The level of the harmonics, that can be viewed on the spectral analyzer screen, is measured from left to right. Measurements shall be made until at least 3 (duty cycle 50%) or 5 (duty cycle 25%) consecutive components are found, below the level of $-40\,\text{dBm}$. The last component indicating a higher value of $-40\,\text{dBm}$ is the last component in the spectrum, situated on frequency $k \cdot 200\ \text{kHz}$. Since the signals are in the base band, the signal bandwidth is $B = k \cdot 200\ \text{kHz}$. If all of the components on the screen are higher than $-40\,\text{dBm}$, the CENTER value is increased, and the level of the harmonics is still measured. If the components have lower values than $-40\,\text{dBm}$, the value indicated by CENTER is decreased.

F) The theoretical and experimental spectral analysis of symmetrical triangular periodic signal.

The waveform generated by the function generator is changed to obtain a triangular signal (Waveforms button). Its frequency is set to $f_0 = 200\ \text{kHz}$, signal symmetry at 50% (DutyCycle button) and amplitude of the triangular signal E so that the fundamental level measured with the spectral analyzer is $0\ \text{dBm}$. For the triangular signal the first 12 spectral components are measured and under these conditions, it is determined the bandwidth occupied by the triangular signal following the steps described above. The experimental results are then completed into a table similar to Table 2, and the theoretical ones will be completed at home.

The E_0 amplitude for the studied signal is measured using the oscilloscope following these steps: press the Auto Setup button, press the Measure button, than All Measure is selected to be On. The channel to which the studied signal was connected is selected as the source, and Max as the type of measurement.

G) Determine the distortion factor.

A sinusoidal signal (Waveforms button), generated by the function generator, having a frequency of $f_0 = 200\ \text{kHz}$ and the level of the fundamental equal to 0 dBm, is applied at the input of the spectrum analyzer. Measure the level of the first 10 spectral components and calculate the distortion factor using the relationship (3).

Repeat the measurements for a sinusoidal signal produced by the function generator, having a fundamental frequency of $f_0 = 200\ \text{kHz}$ and a reference level of 10 dBm.

H) Repeat point F) for a triangular signal with $f_0 = 10\ \text{kHz}$, this time using the oscilloscope Siglent SDS 1202X-E for the frequency domain.

The Siglent SDS 1202X-E oscilloscope can also be used as a spectral analyzer. In order to enter in the spectral analysis mode, a signal is connected to the input of one of the two channels, Auto Setup button is pressed, then the Math menu button is pressed, FFT (Fast Fourier Transform) is selected and at Source is selected the channel on which the signal is connected. Use the rotating button from section Horizontal modify sec/div to set span to 100ms/div.

For measurements in the frequency domain the oscilloscope is used. The reference voltage is determined by applying to the function generator a sinusoidal signal with the frequency $10\,\text{kHz}$, the amplitude of which is adjusted so that the amplitude of the fundamental component, viewed on the oscilloscope, used as a spectrum analyzer to be of 0 dB. Use the cursors (Cursor button), select Source - MATH for the measurement of frequencies (by pressing the button corresponding to X underneath the screen) or amplitudes (by pressing the button corresponding to Y underneath the screen). The fundamental frequency will be determined with the cursor X1 or X2. To move the cursors on the screen, use the rotating button Intensity/Adjust. Then commute to Y and cursor Y1 or Y2 are placed at 0 dBV. The effective value of this signal will be the reference voltage. Measure the level of all the other spectral components using cursor Y1 or Y2 by setting the cursor on their tip. For identifying the order of the measured spectral component, use the X cursors setting X1 and X2 on the respective component and reading the frequency indicated by the used cursor.

I) The spectra of theoretical and experimental amplitudes for the studied signals are plotted on a millimetre paper, $\left.\frac{A_k}{A_1}\right|_{\text{teoretic}}$ and $\left.\frac{A_k}{A_1}\right|_{\text{experimental}}$, depending on the frequency.

For the same duty cycle, the spectra are plotted on the same graph (the theoretical value through a segment, and the experimental value by one point, using a different colour for segments and another for points).

What connections can you see between the rise time measured at point D) and the signal's spectrum?

For the triangular signal, the theoretical and experimental amplitude spectra are plotted on another millimetre paper, $\left.\frac{A_k}{A_1}\right|_{\text{teoretic}}$ and $\left.\frac{A_k}{A_1}\right|_{\text{experimental}}$, on a frequency domain, on the same coordinate axes.

J) Determine the power dissipated on a 1Ω resistor for the rectangular signals based on the amplitude spectrum measured in Table 2 using equation (12).

Note: While computing the power, using relation (12), mind the following aspect: using the spectrum analyzer the effective values of the amplitudes A_k $A_{k,ef}$ where $A_{k,ef} = \frac{A_k}{\sqrt{2}}$.

Compare the power obtained using the experimental data P_e and the power of the fundamental P_1, with the power calculated based on the time expression, P_t, equation (13). The $\frac{P_e}{P_t}$ and $\frac{P_1}{P_t}$ ratios are determined.

Note: The amplitudes E_1 and E_2 from relation (13) are measured with the oscilloscope and are given by the formula $E_i = E_{0i} \cdot \theta, i \in \{1,2\}$, where E_{01} and E_{02} are the maximum, respectively, the minimum peak amplitudes of the rectangular measured signal, measured with the oscilloscope and θ is the division coefficient determined in point A).

K) Determine the power of the triangular signal based on the measured components, relation (6) (mind the observations from point J))

The P_e is compared with P_t power which is calculated using the equation (7). The $\frac{P_e}{P_t}$ and $\frac{P_1}{P_t}$ ratios are determined, where P_1 is the power of the fundamental component.

1.4. Preparatory Questions

a) Determine $A_1[\text{V}]$ and $A_2[\text{dBm}]$ using $A_1 = 20\ \text{dBm}$ and $A_2 = 0{,}01A_1$, $U_{ref} = 0{,}2236\ \text{V}$. Repeat for $U_{ref} = 0{,}775\ \text{V}$.

b) If $A_1 = 20\ \text{dBm}$ and $A_2 = 0{,}01A_1$, what is the difference (in dB) between $A_1[\text{dBm}]$ and $A_2[\text{dBm}]$? Repeat for $A_2 = 0{,}1A_1$ and $A_2 = 0{,}001A_1$. What do you observe?

c) If $A_1 = 20\ \text{dBm}$ and $A_2 = 14\ \text{dBm}$, which is the value of the $\frac{A_2}{A_1}$ ratio in level units. Specify the unit of measurement.

d) Determine the power dissipated by a harmonic sinusoidal signal with a level of $0\,\text{dBm}$ ($U_{ref} = 0{,}2236\ \text{V}$) on a resistor of $50\,\Omega$, $75\,\Omega$, $600\,\Omega$. Repeat for a level signal of $10\,\text{dBm}$. What power increase determines the change with $10\,\text{dB}$ the signal level?

e) Determine the power dissipated by a harmonic signal with a level of $0\,\text{dBm}$ ($U_{ref} = 0{,}775\ \text{V}$) on a resistor of $50\,\Omega$, $75\,\Omega$, $600\,\Omega$. Repeat for a level signal of $10\,\text{dBm}$. What power increase determines the change with $10\,\text{dB}$ the signal level?

f) Determine the damping factor for the resistive divider from figure 8 if R - 50Ω (the typical input impedance of a signal analyzer) respectively R = 1MΩ (the typical input impedance of an oscilloscope). What do you observe? What will be the peak value of the signal U_2 if the signal U_1 is sinusoidal with the effective value of $\sqrt{2}\,[\text{V}]$?

g) Draw the amplitude and phase spectra for the signal $s(t) = 2 + 2\sin(100t + \pi) - 3\cos(200t) + \cos^2\left(400t - \frac{\pi}{4}\right)$.

h) A periodic signal was measured with a spectrum analyzer. The following values were obtained: $A_1 = 20\ \text{dBm}$, $A_2 = 10\ \text{dBm}$, $A_3 = -25\ \text{dBm}$, $A_4 = 1\ \text{dBm}$, $A_5 = -21\ \text{dBm}$, $A_6 = -25\ \text{dBm}$ and $A_7 = -30\ \text{dBm}$. Determine the effective bandwidth of the signal if the limit is $0{,}01A_1$, $0{,}1A_1$, respectively $0{,}001A_1$.

1.5. Questions

a) What is the value of the DC offset for the studied signal in B and C?

b) What is the rise time for an ideal rectangular signal?

c) Why can we not obtain a perfect extinction (suppression) of the even harmonics when $\tau/T = 1/2$?

d) Two rectangular periodic signals have the same period and complementary duty factors $\tau_1/T + \tau_2/T = 1$. What is the relationship between the amplitudes A_k of the two signals?

1.6. Exercises

a) Adjust the parameters of a periodic rectangular signal such that $T = 50\,\mu\text{s}$, $\tau/T = 1/3$, $A = U_r$. Determine the values of the amplitudes A_k, $k = 0$.

b) When a sine signal was measured, the level of the harmonics were the following: $n_1 = -3\,\text{dB}$, $n_2 = -43\,\text{dB}$, $n_3 = -49\,\text{dB}$, $n_4 = -63\,\text{dB}$ $(U_{ref} = 1\,\text{V})$. Determine the harmonic of 1st order (in mV) and the distortion factor.

c) In the spectral analysis of a periodic rectangular signal, it was found that the 2nd order harmonic has with 25 dB less than the fundamental. What duty cycle does the analyzed signal have? What will be the difference in dB between the 1st order harmonic and the 3rd harmonic level for this signal?

d) The signal in a) is applied at the input of an ideal low pass filter with the cut-off frequency $f_t = 45\,\text{kHz}$. Make a graphical representation of the output signal