The third laboratory

FREQUENCY MODULATED SIGNALS WITH HARMONIC CARRIER SIGNAL

3.1. Objective

In this laboratory, the spectral analysis of frequency modulated oscillations with sinusoidal modulator signal, rectangular signal and triangular signal will be studied.

3.2. Theoretical aspects

It is known that the general expression of an oscillating modulation with a sinusoidal modulator signal is:

$$x(t) = A(t) \cdot \cos \Phi(t), \tag{1}$$

where $A(t)$ is the amplitude of the oscillation, $\Phi(t)$ is the instantaneous phase and $\Omega(t) = \dfrac{d\,\Phi(t)}{d\,t}$ is the instantaneous frequency. In the absence of the modulator signal, $x_m(t)$, the parameters of the oscillation have the following expressions:

$$\begin{aligned} A(t) &= A_0 = const., \\ \Omega(t) &= \Omega_0 = const., \\ \Phi(t) &= \int \Omega(t)\,dt + \varphi_0 = \Omega_0 t + \Phi_0, \end{aligned} \tag{2}$$

and the relation (1) is reduced to the expression of the carrier signal:

$$x_0(t) = A_0 \cos(\Omega_0 t + \Phi_0). \tag{3}$$

In the case of a frequency modulated signal, the instantaneous amplitude $A(t)$ is constant and it is equal with A_0, the instantaneous frequency $\Omega(t)$ varies around the carrier frequency Ω_0, in the rhythm of the modulator signal $x_m(t)$, following a linear law:

$$\Omega(t) = \Omega_0 + K_F x_m(t), \tag{4}$$

where K_F is a specific constant to the FM modulator.

By integrating the relationship (4), the expression of the instantaneous phase is obtained:

$$\Phi(t) = \Omega_0 t + K_F \int_0^t x_m(\tau)\, d\tau + \Phi_0. \tag{5}$$

In these conditions, we obtain the general expression of a frequency modulated signal with a harmonic carrier signal and a random modulating signal:

$$x_{MF}(t) = A_0 \cos\left[\Omega_0 t + K_F \int_0^t x_m(\tau)\, d\tau + \Phi_0\right] \tag{6}$$

If the modulating signal is harmonic:

$$x_m(t) = A_m \cos(\omega_m t + \varphi_m), \tag{7}$$

relations (4) and (5) become:

$$\Omega(t) = \Omega_0 + \Delta\Omega \cos(\omega_m t + \varphi_m), \tag{8}$$

$$\Phi(t) = \Omega_0 t + \frac{\Delta\Omega}{\omega_m} \sin(\omega_m t + \varphi_m) + \Phi_0, \tag{9}$$

where $\Delta\Omega = K_F A_m$ is called the frequency deviation of the FM signal and represents the maximum variation of the instantaneous frequency $\Omega(t)$, related to Ω_0, as it is shown in Figure 1.

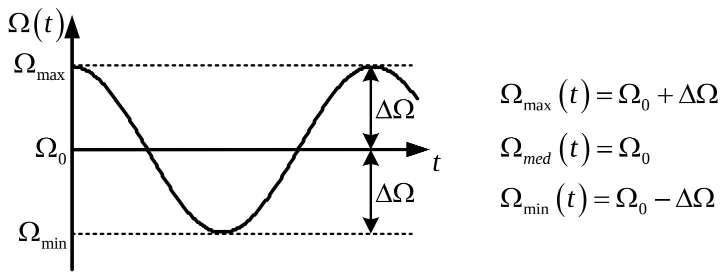

Figure 1. Graphic representation of instantaneous frequencies for a FM signal with harmonic carrier signal and harmonic modulator signal

Ratio

$$\beta = \frac{\Delta\Omega}{\omega_m} = \frac{K_F A_m}{\omega_m} \tag{10}$$

is called a *frequency modulation index*. In this case, the expression of the frequency modulated signal is:

$$x_{MF}(t) = A_0 \cos\left[\Omega_0 t + \beta \sin\left(\omega_m t + \varphi_m\right) + \Phi_0\right] \tag{11}$$

Figure 2 shows waveforms for $x_m(t)$, $x_0(t)$ and $x_{MF}(t)$ signals.

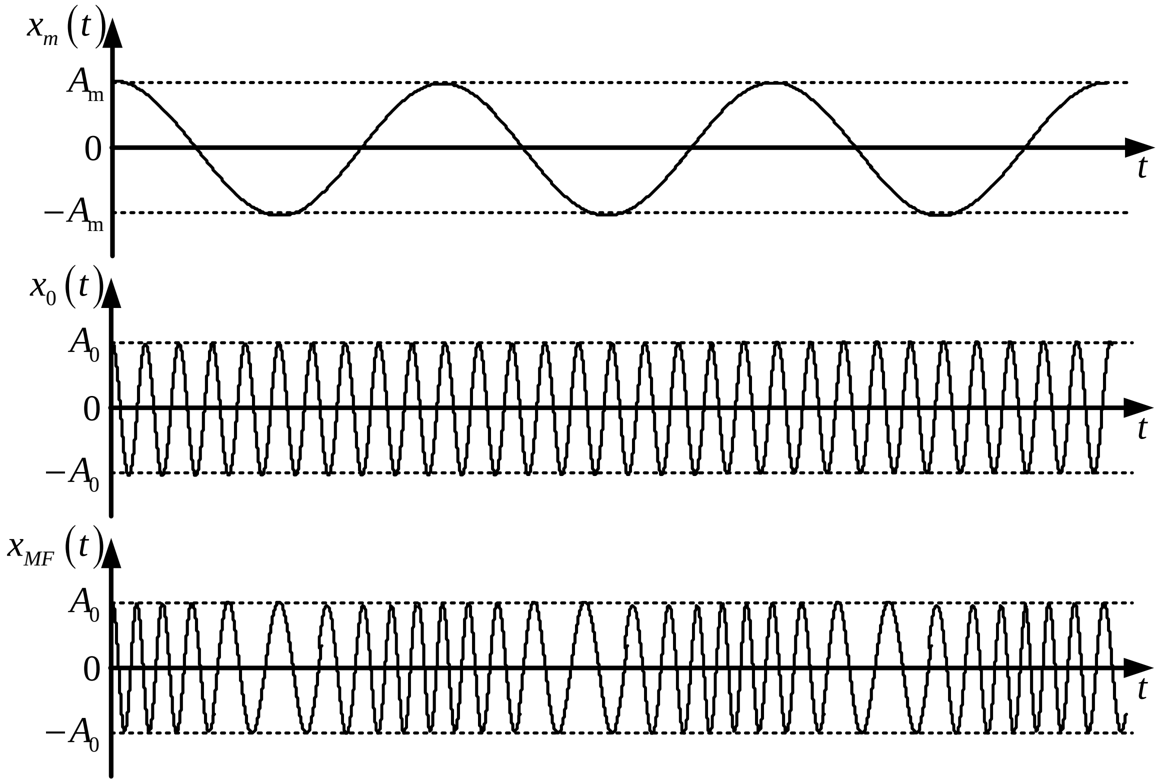

Figure 2. Waveforms for modulator, carrier and frequency modulated signals in the harmonic case

For the spectral analysis of the $x_{MF}(t)$ signal, it is used the following relation:

$$e^{j\beta \sin z} = \sum_{k=-\infty}^{+\infty} J_k(\beta) e^{jkz}, \tag{12}$$

where $J_k(\beta)$ are Bessel's first-order functions, by k order and β argument. In Figure 3 are shown some graphs of Bessel's first-order functions, by k order and β argument.

Table 1 presents the values of β, for which the cancellation for the Bessel's first-order function, by k order and β argument, is obtained.

Table 1 β values for which $J_k(\beta)=0$

k	β			
0	2,40	5,52	8,65	11,79
1	3,83	7,02	10,17	13,32
2	5,14	8,42	11,62	14,80
3	6,38	9,76	13,02	16,22
4	7,59	11,06	14,37	17,62

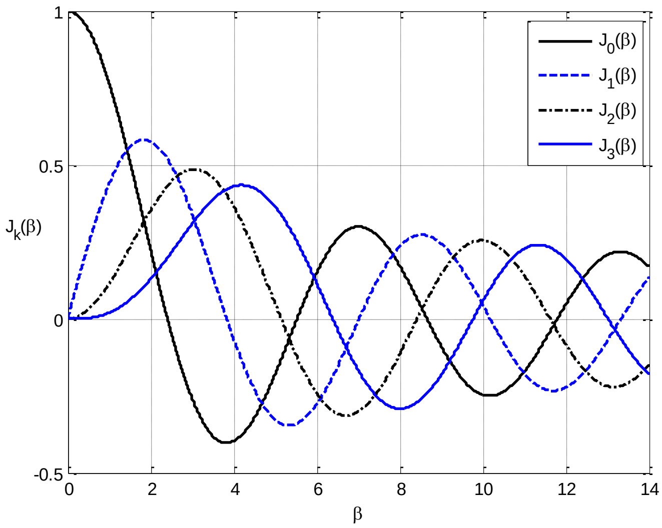

Figure 3. Graphic representation of the $J_0(\beta)$, $J_1(\beta)$, $J_2(\beta)$ and $J_3(\beta)$ functions

Among the most important properties of Bessel's first-order functions, by k order and β argument, are mentioned:

$$J_k(\beta)=(-1)^k J_{-k}(\beta), \tag{13}$$

$$\sum_{k=-\infty}^{+\infty} J_k^2(\beta)=1, \tag{14}$$

$$J_{k+1}(\beta)+J_{k-1}(\beta)=\frac{2k}{\beta}J_k(\beta). \tag{15}$$

Using the development (12), the relation (11) can be written:

$$x_{MF}(t)=A_0\sum_{k=-\infty}^{+\infty} J_k(\beta)\cos\left[(\Omega_0+k\omega_m)t+\Phi_0+k\varphi_m\right] \tag{16}$$

The graphical representation of the amplitude spectral diagram for the frequency modulated signal is shown in Figure 4.

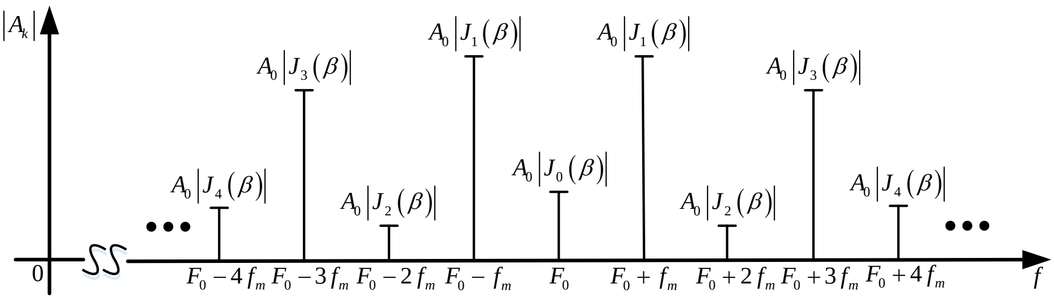

Figure 4. The amplitude spectral diagram of the frequency modulated signal with harmonic carrier signal and harmonic message signal

Expression (16) has an infinity of terms and the signal bandwidth is infinite. In practice, it is limited to N terms:

$$x_{MF}(t) \cong A_0 \sum_{k=-N}^{+N} J_k(\beta)\cos\left[(\Omega_0 + k\omega_m)t + \Phi_0 + k\varphi_m\right] \tag{17}$$

and for energy considerations, it is found $N = \beta + \sqrt{\beta} + 1$, meaning:

$$B_{MF} = 2\left(\beta + \sqrt{\beta} + 1\right) f_m. \tag{18}$$

If $\beta >> 1$, 1 and $\sqrt{\beta}$ can be neglected and the bandwidth becomes:

$$B_{MF} \cong 2\beta f_m = 2\Delta F. \tag{19}$$

The relation (19) is remarkable because for $\beta >> 1$ the bandwidth does not depend on the frequency of the modulator signal but only on the maximum frequency deviation.

Relation (11) it can also be written in the following form:

$$\begin{aligned} x_{MF}(t) = A_0 \cos(\Omega_0 t + \Phi_0)\cos\left(\beta \sin(\omega_m t + \varphi_m)\right) - \\ - A_0 \sin(\Omega_0 t + \Phi_0)\sin\left(\beta \sin(\omega_m t + \varphi_m)\right). \end{aligned}$$

If $\beta \sin(\omega_m t + \varphi_m) << \dfrac{\pi}{2} \Leftrightarrow \beta < 0{,}5$ the following approximations can be used:

$$\begin{cases} \cos\left(\beta \sin(\omega_m t + \varphi_m)\right) \cong 1 \\ \sin\left(\beta \sin(\omega_m t + \varphi_m)\right) \cong \beta \sin(\omega_m t + \varphi_m) \end{cases}, \tag{20}$$

which are also called the narrowband approximation. In this way, the expression of a narrowband FM signal is obtained:

$$x_{MF}(t) = A_0 \cos(\Omega_0 t + \Phi_0) - \frac{\beta A_0}{2} \cos\left[(\Omega_0 - \omega_m)t + \Phi_0 - \varphi_m\right] + \\ + \frac{\beta A_0}{2} \cos\left[(\Omega_0 + \omega_m)t + \Phi_0 + \varphi_m\right] \tag{21}$$

The graphical representation of the amplitude spectrum is shown in Figure 5.

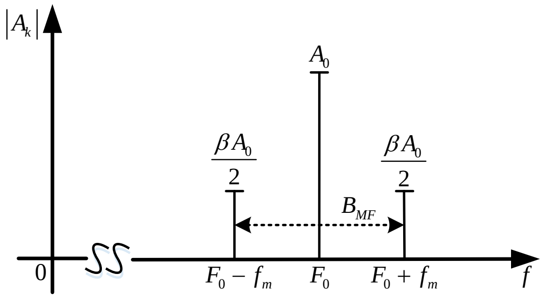

Figure 5. The amplitude spectrum of a narrowband FM signal

Figure 5 shows that the bandwidth of the FM signal is:

$$B_{MF} = 2 f_m, \tag{22}$$

and the upper side component module is equal to the lower side component module.

The dissipated power of a FM signal with a sinusoidal modulator signal on a $\mathrm{R} = 1\Omega$ resistor has the expression:

$$\mathrm{P}_{MF} = U_{ef}^2 = \left(\frac{A_0}{\sqrt{2}}\right)^2 \sum_{k=-\infty}^{+\infty} J_k^2(\beta) = \frac{A_0^2}{2} \tag{23}$$

From the above relationship, it results that the effective value of the FM signal has the expression:

$$U_{ef} = \frac{A_0}{\sqrt{2}} \tag{24}$$

3.3. Practical part

The connections from figure 6 are realized.

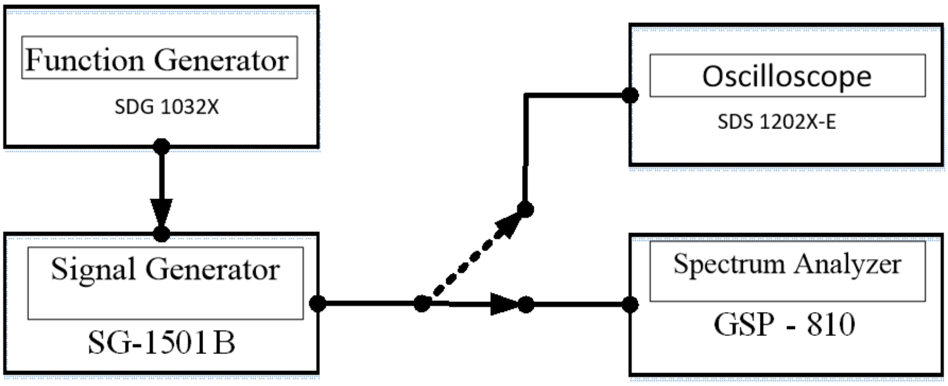

Figure 6. The measurement scheme used in the laboratory

A) Determination of the modulator characteristic, $\beta(A_m)$, for a frequency deviation equal to 15 kHz

The characteristic of the modulator is built on the basis of the method of carrier extinctions, based on the variation of the $J_0(\beta)$ function (see Figure 3). In Fig. 4 we can see that the amplitude component of the carrier signal is given by the relation $A_0|J_0(\beta)|$. β values for which $J_0(\beta)=0$ (the carrier component is cancelled) are given in Table 1, and four of them, enough for lifting the characteristic, have been selected and included in Table 2.

To draw the MF modulator characteristic, it is proceed as it follows:

1. The parameters of the carrier signal and of the modulation process are adjusted as it follows: proceed so that only the FM and EXT buttons to be activated in the MODULATION buttons group of the modulated signal generator (LEDs on). Also, we should set the carrier frequency: press the FREQ button from the DATA ENTRY button grouping (attention: the FREQ button of the modulated signal generator, not the button of the signal generator!), enter the 1000 value using the numeric keypad and press the kHz key. The carrier frequency is set to 1 MHz, and the FREQUENCY display shows the value of 1.000.0 MHz, the first point being a decimal point, and the second one being just a decimal separator to ease the reading.

2. Adjust the frequency deviation (corresponding to the $\Delta\Omega$ parameter) to 15 kHz as it follows: press the MOD button (located below the FREQ button previously used), enter the 15 value using the same keypad as in the previous operation and press the ENT button that should have the LED on.

3. Connect the output of the modulated signal generator (OUTPUT) to the input (RF INPUT 50Ω) of the spectral analyzer. Adjust the parameters of the spectral analyzer: center the screen around the carrier frequency (1 MHz) by pressing the CENTER button, entering the 1 value and pressing the MHz button. Adjust the SPAN parameter to 5 kHz / div by pressing the SPAN button and using the rotary knob to select the desired value. Adjust the reference level to 10 dBm by pressing the REF LVL button and using the rotary knob to select the desired value.

4. It is desired to adjust the level of the carrier component to 0 dBm. As it can be seen in Figure 4, the amplitude of the carrier component depends on $J_0(\beta)$ which obviously depends on the modulation index β which, in turn, it depends on the A_m signal amplitude. This initial setting of the amplitude of the carrier component is made under the conditions $J_0(\beta)$=1, therefore, according to Figure 3, under conditions β=0, so A_m = 0. Therefore, during this subpoint from the practical part, the function generator is disconnected from the input of the modulated signal generator. Next, measure the carrier with the spectral analyzer using a cursor by pressing the MKR button, entering the 1 value and pressing the MHz button. Modify the carrier component level from the modulated signal generator until it becomes 0 dBm, measured with the spectral analyzer (attention, not displayed on the modulated signal generator, but measured with the analyzer!) in the following way: underneath one of the digits displayed on a display (EXT MODULATION, FREQUENCY or OUTPUT LEVEL), a green LED will light up to indicate that the digit is selected. This LED will be moved in the OUTPUT LEVEL display by pressing repeatedly a double arrow (for example ▶▶). Then, select the least significant digit on that display by pressing repeatedly the single arrow (for example ▶). Use the rotary knob underneath the arrow group to change the carrier level. (Because the input impedance in the spectral analyzers is no longer 50 Ω, the value displayed on the

OUTPUT LEVEL display of the modulated signal generator will probably differ from 0 dBm, generally being higher).

5. Connect one of the main outputs of the function generator to the input of the modulated signal generator (EXT INPUT AF/L). Set the message signal parameters from the function generator: sinusoidal waveform (Waveforms button and then the button below the screen corresponding to the desired waveform), adjust its frequency to 10 kHz (the Frequency button below the screen is pressed until Frequency is selected in blue). The effective value (rms) is a parameter that will vary within the experiment (the Amplitude button below the screen is pressed until Amplitude is selected in blue). The amplitude of the modulator signal $A_m\left[V_{rms}\right]$ is changed starting from zero (or the smallest possible value) to the maximum value, modifying the least significant digit using the rotary knob. Table 2 is filled in with the effective values of the message signal for which the carrier component measured by the spectral analyzer becomes less than –40 dBm (phenomenon called carrier extinction). Three consecutive extinctions are searched for, and the effective values of the modulator signal for which these extinctions occur are written in Table 2. Considering that the function generator is designed to deliver the displayed parameters in a load impedance of 50 Ω and the input impedance of the modulated signal generator is 10 kΩ, the doubled value of the displayed value on the generator screen will be listed in the table (more details are found in the amplitude modulated signal laboratory).

Table 2. Determination of FM modulator characteristic

$A_m\left[V_{rms}\right]$	0			
β	0	2,40	5,52	8,65

B) Draw the modulator characteristic

By the results from table 2, we build the characteristic of the modulator $\beta = f\left(A_m\right)$. The K_F is determined from the graphic, considering the relation (10).

C) Determination of the modulator characteristic, $\beta(A_m)$, for the frequency deviation equal to 60 kHz

By adjusting the frequency deviation, from the modulated signal generator to 60 kHz (proceeding as in point A, subpoint 2), points A and B are repeated, drawing the new characteristic on the same graphic.

How do you interpret the two charts and what relationship exists between their slopes?

D) Spectral measurements for sinusoidal modulator signal and $\beta = 0{,}3$

Returning to the 15 kHz frequency deviation value, read the value of the modulating signal voltage from the modulator characteristic graphic $A_m\left[V_{rms}\right]$, for which it is obtained $\beta = 0{,}3$. Fix half of this value to the function generator and measure the spectral values $\left(C_N = A_0 \cdot \left|J_N(\beta)\right|\right)$ with the spectral analyzer for $N = -3, -2, -1, 0, 1, 2, 3$.

Table 3. Determination of spectral components for FM signal

N	-3	-2	-1	0	1	2	3
$f\left[\mathrm{MHz}\right]$	0.97	0.98	0.99	1	1.01	1.02	1.03
$C_N\left[\mathrm{dBm}\right]$							

E) Spectral measurements for a triangular and rectangular message signal when $\beta = 0{,}3$

Repeat point D, modifying the shape of the modulator signal in the function generator. Consider triangular and rectangular signal cases for the same $A_m\left[V_{rms}\right]$ values as for point D. Tables 4 and 5 will be completed in similar way to point D.

F) Spectral measurements for sinusoidal modulator signal and $\beta = 1$

Repeat point D, for the sinusoidal modulator signal and for $\beta = 1$ (A_m will be extracted from the graph corresponding to the 15 kHz frequency deviation). Table 6 will be filled in (experimental data).

Table 6. Determination of the spectral components for the MF signal with $\beta = 1$

N	-3	-2	-1	0	1	2	3
$f\,[\mathrm{kHz}]$							
$C_N^{\mathrm{experimental}}\,[\mathrm{dBm}]$							
$C_N^{\mathrm{experimental}}\,[\mathrm{V}]$							
$C_N^{\mathrm{theoretical}}\,[\mathrm{V}]$							

G) Spectral measurements for sinusoidal modulator signal and $\beta = 4$.

Repeat point F for $\beta = 4$. Table 7 will be completed in a similar way.

Table 7. Determination of the spectral components for the MF signal with $\beta = 4$

N	-8	-7	-6	...	6	7	8
$f\,[\mathrm{kHz}]$							
$C_N\,[\mathrm{dBm}]$							

H) Spectral measurements for sinusoidal modulator signal and $\beta = 9$

For $\beta = 9$, measure the spectral components, C_N , using spectrum analyzer for $N = -14, -13, 12, 0, 12, \ 13, 14$ and fill in Table 8.

Table 8. Determination of the spectral components for the MF signal with $\beta = 9$

N	-14	-13	-12	0	12	13	14
$f\,[\mathrm{kHz}]$							
$C_N\,[\mathrm{dBm}]$							

I) The bandwidth of frequency modulated signal generator is measured.

For this, the sinusoidal signal with the frequency $f_m = 10\,\mathrm{kHz}$ is fixed at the function generator. A frequency deviation of $60\,\mathrm{kHz}$ is set at the modulated signal generator. On the spectrum analyzer, the central frequency (CENTER) is set to $1\,\mathrm{MHz}$ and the SPAN value to $100\,\mathrm{kHz/div}$ by using the SPAN key and then the RBW key is fixed to $30\,\mathrm{kHz}$. Increase the level of the modulator signal,

A_m, until it is observed on the spectrum analyzer that the MF bandwidth does not change. Using markers, determine the frequencies F_1 and F_2, which are corresponding to the margins of the MF signal generator bandwidth.

J) Investigating the FM radio band using the spectral analyzer

An antenna is required from the teacher. Connect the antenna to the input of the spectral analyzer (RF INPUT 50Ω). The following parameters are set for viewing the FM radio (88 MHz - 108 MHz): CENTER 90 MHz, SPAN 5 MHz / div, press the MKR button, enter 88 and press the MHz key. Press ENTER, enter the 108 value and press the MHz key. The FM radio band is founded between the two cursors, on the analyzer screen. Choosing one of the two markers, we determine the frequencies where the amplitude of the spectral components is greater than −35 dBm (about 4 components). Using the Internet, we investigate what radio stations are emitted on the specified frequencies.

The center is set at one of the specified frequencies (89 MHz or 102 MHz is recommended). Activate the FM demodulator by pressing the SHIFT key, then the CENTER key. Select DEMOD TYPE: WIDE using the rotary knob. Rise the sound level using the VOL potentiometer located below the spectral analyzer screen. Change the center frequency slightly (by pressing the CENTER button, pressing the analyzer key ”▶ ” until the digit corresponding to the tens of kHz is selected and using the rotary button to change it). What's to be observed?

K) Draw the amplitude spectrum on the millimeter paper and determine the bandwidth of the FM signal using the data from point D.

L) Using the value of the carrier spectral component measured at point D, determine A_0 knowing that $C_0 = A_0 \cdot J_0(0{,}3)$.

With the value A_0 previously determined, calculate: $C_1^{\text{theoretical}}$, $C_2^{\text{theoretical}}$ and $C_3^{\text{theoretical}}$ knowing that:

k	0	1	2	3
$J_k(0{,}3)$	0,9776	0,1483	0,0112	0,000559

M) Draw the amplitude spectrum on the millimeter paper for the both signals from point E and determine the bandwidth of the FM signals.

How do you explain the results obtained at points D and K?

N) For the calculation of $C_N^{\text{experimental}}[\text{V}]$, use the data from table 6 and the relation:

$$C_N^{\text{experimental}}[\text{V}] = U_{ref} \cdot 10^{\frac{A_N^{\text{experimental}}[\text{dBm}]}{20}}, \tag{25}$$

where U_{ref} is the effective value of the reference voltage $U_{ref} = 0{,}2236\,\text{V}$.

Using the value of the carrier spectral component $\left(C_0 = A_0 \cdot |J_0(1)|\right)$ measured at point F and knowing that $J_0(1) = 0{,}7652$ and $J_1(1) = 0{,}4401$, determine A_0. Using the A_0 value previously determined and using the recurrence relation between Bessel's first-order functions (15), determine $C_N^{\text{theoretical}}$ with the relation:

$$C_N^{\text{theoretical}} = A_0 \cdot \left|J_N(1)\right|$$

How much is the FM bandwidth? Compare the obtained value with the theoretical one obtained with the relationship (18).

O) Using the values determined at the point F, calculate the power of the FM modulated signal and then verify the relation (23) by taking only the spectral components measured in Table 6.

P) Determine the bandwidth of the modulated signal studied at the point G. How do you explain the results obtained?

R) How much is the bandwidth of the modulated signal studied at the point H?

3.4. Questions

a) How should the ideal characteristic look $\beta\left(A_m\right)$ and what significance the deviation has from the ideal form?

b) How the bandwidth occupied by the FM signal will be changed if A_m is maintained constant and f_m varies?

c) By what processes it can be pointed out that a FM generator also produces a parasitic amplitude modulation?

3.5. Applications

a) Determining the characteristic of a FM modulator using the method of extinction of the carrier, the first cancellation of the component carrier it is found at $A_m = 1\,\mathrm{V}$. The A_m is decreased at $0{,}1\,\mathrm{V}$. Represent the amplitude spectrum knowing that $A_0 = 0\,\mathrm{dB}$ and $U_{ref} = 0{,}223\,\mathrm{V}$.

b) Calculate the parameters of a FM signal with harmonic carrier signal and harmonic message signal if $A_0 = 0\,\mathrm{dB}$, $F_0 = 1000\,\mathrm{kHz}$, $f_m = 10\,\mathrm{kHz}$, $\Delta F = 10\,\mathrm{kHz}$. Calculate and draw the amplitude spectrum for this signal.

c) Calculate the parameters of a FM signal with harmonic carrier signal and harmonic message signal if $A_0 = 0\,\mathrm{dB}$, $F_0 = 1000\,\mathrm{kHz}$, $f_m = 10\,\mathrm{kHz}$, $\Delta F = 3\,\mathrm{kHz}$. Calculate and draw the amplitude spectrum for this signal.