MATRIX PARAMETERS OF THE TWO-PORT NETWORKS

1) The objective of the laboratory work

The paper studies the methods of experimental measurement of the parameters associated with some parameters for describing drifts, as well as the relationships between these parameters.

2) Theoretical aspects

a) Electric two-port network. Associated parameters

A two-port network consists of an electrical network accessible at four terminals in such a way that the terminals are grouped into two pairs (1 – 1') and (2 – 2'), and the current entering one terminal of a pair (1 or 2) is equal to the current coming out of the other terminal of the pair (1' or 2').

A two-port network is characterized by:

- The two currents I_1 și I_2;
- The two port voltages U_1 și U_2.

By capital letters we denote complex amplitudes as a function of ω or s. Check Figure 1.

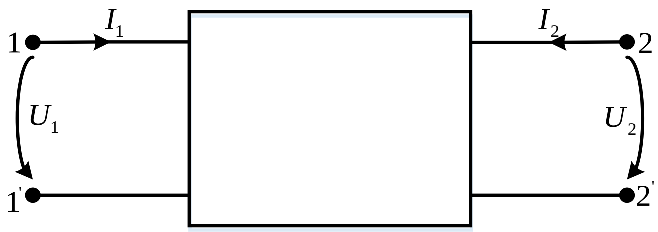


Figure 1

The four quantities U_1, U_2, I_1 and I_2 can be uniquely determined from a set of four linearly independent non-homogeneous equations. Since two boundary conditions (terminals) can be established for the two ports, a set of two more equations (homogeneous or not) are needed between the four quantities. These two equations, which solely depend on the internal structure of the two-port network, constitute what is called a formalism for describing the two-port network. If the two-port network is linear, then a formalism makes a linear application from two of the four quantities (U_1, U_2, I_1, I_2) to the other two quantities. Since the order within a pair is not essential, it follows that for a two-port network, there are:

$$n = C_4^2 = \frac{4 \cdot 3}{1 \cdot 2} = 6 \text{ formalisms}$$

Noting with X_1, X_2 as the first two elements of a formalism, and with X_3, X_4 as the other two, it follows from the given definition that:

$$M: (X_1, X_2) \rightarrow (X_3, X_4)$$

$$\begin{bmatrix} X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad (1)$$

where the M_{ij} parameters are generally complex numbers.

Each of the 6 formalisms comprises a unique set of parameters:

- Z or impedance parameters;
- Y or admittance parameters;
- A or chain or transmission parameters;
- B or reverse chain or reverse transmission parameters;
- h or hybrid parameters;
- g or inverse hybrid parameters.

The structures of the six formalisms are shown in Table 1. The determinant of the matrix characterizing the considered parameters was noted $\Delta_{\boldsymbol{M}}$.

Tabel 1. Different formalisms for describing two-port networks:

Z-impedance parameters $\begin{bmatrix}U_1\\U_2\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12}\\Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}I_1\\I_2\end{bmatrix}$ $\begin{bmatrix}U_1\\U_2\end{bmatrix}=[\boldsymbol{Z}]\begin{bmatrix}I_1\\I_2\end{bmatrix}$	**Y**-admittance parameters $\begin{bmatrix}I_1\\I_2\end{bmatrix}=\begin{bmatrix}Y_{11} & Y_{12}\\Y_{21} & Y_{22}\end{bmatrix}\begin{bmatrix}U_1\\U_2\end{bmatrix}$ $\begin{bmatrix}I_1\\I_2\end{bmatrix}=[\boldsymbol{Y}]\begin{bmatrix}U_1\\U_2\end{bmatrix}$
A-chain parameters $\begin{bmatrix}U_1\\I_1\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12}\\A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}U_2\\-I_2\end{bmatrix}$ $\begin{bmatrix}U_1\\I_1\end{bmatrix}=[\boldsymbol{A}]\begin{bmatrix}U_2\\-I_2\end{bmatrix}$	**B**-reverse chain parameters $\begin{bmatrix}U_2\\I_2\end{bmatrix}=\begin{bmatrix}B_{11} & B_{12}\\B_{21} & B_{22}\end{bmatrix}\begin{bmatrix}U_1\\-I_1\end{bmatrix}$ $\begin{bmatrix}U_2\\I_2\end{bmatrix}=[\boldsymbol{B}]\begin{bmatrix}U_1\\-I_1\end{bmatrix}$
h-hybrid parameters $\begin{bmatrix}U_1\\I_2\end{bmatrix}=\begin{bmatrix}h_{11} & h_{12}\\h_{21} & h_{22}\end{bmatrix}\begin{bmatrix}I_1\\U_2\end{bmatrix}$ $\begin{bmatrix}U_1\\I_2\end{bmatrix}=[\boldsymbol{h}]\begin{bmatrix}I_1\\U_2\end{bmatrix}$	**g**-reverse hybrid parameters $\begin{bmatrix}I_1\\U_2\end{bmatrix}=\begin{bmatrix}g_{11} & g_{12}\\g_{21} & g_{22}\end{bmatrix}\begin{bmatrix}U_1\\I_2\end{bmatrix}$ $\begin{bmatrix}I_1\\U_2\end{bmatrix}=[\boldsymbol{g}]\begin{bmatrix}U_1\\I_2\end{bmatrix}$

Inversion relationships are observed:

$$[\boldsymbol{Z}]^{-\mathbf{1}}=[\boldsymbol{Y}],\qquad [\boldsymbol{h}]^{-\mathbf{1}}=[\boldsymbol{g}]$$

Parameters M_{ij} associated with a particular formalism have physical meanings that lead to methods of measurement. For example, from the equation $\mathrm{X}_3=M_{11}\mathrm{X}_1+M_{12}\mathrm{X}_2$ results:

$$M_{11}=\left.\frac{X_3}{X_1}\right|_{X_2=0}\qquad M_{12}=\left.\frac{X_3}{X_2}\right|_{X_1=0} \tag{2}$$

Bearing in mind that the quantities $\mathrm{X_i}$ are voltages or currents, it follows that the parameters M_{ij} are resistances, reactances or impedances or transfer functions evaluated under open-circuit or short-circuit conditions.

The conversion relationships between one set of parameters and another are contained in Table 2.

Reciprocity is an important property of RLC networks and is manifested by the existence of a connection between the M_{ij} parameters. In Table 2, in the "Reciprocity" column, the connections imposed by reciprocity in different parameters can be observed.

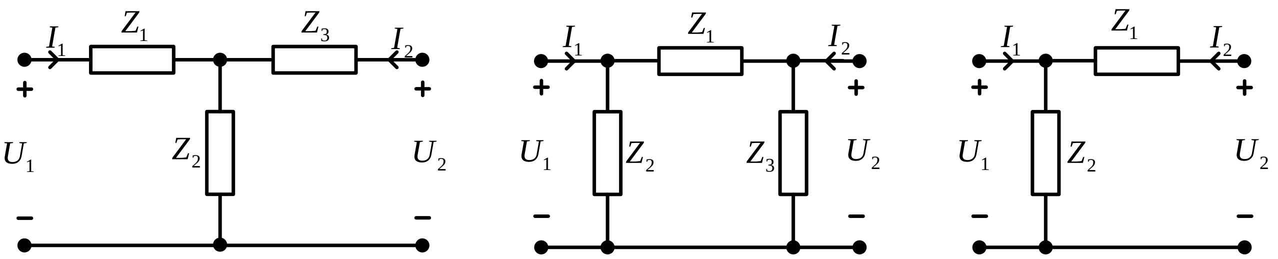
The symmetry of a two-port network introduces a new relationship between the parameters of a two-port network. In Table 2, the last column contains the connections imposed by symmetry in different parameters.

Tabel 2. The relationships between the parameters. $\Delta X = det[X]$

	Z	**Y**	**h**	**A**	Reciprocity	Symmetry
Z	$[\boldsymbol{Z}]$	$\begin{bmatrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{H_{22}} & \frac{h_{12}}{H_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{A_{11}}{A_{21}} & \frac{\Delta A}{A_{21}} \\ \frac{1}{A_{21}} & \frac{A_{22}}{A_{21}} \end{bmatrix}$	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$\begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$	$[\boldsymbol{Y}]$	$\begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{A_{22}}{A_{12}} & -\frac{\Delta A}{A_{12}} \\ \frac{1}{A_{12}} & \frac{A_{11}}{A_{12}} \end{bmatrix}$	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
H	$\begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$	$[\boldsymbol{h}]$	$\begin{bmatrix} \frac{A_{12}}{A_{22}} & \frac{\Delta A}{A_{22}} \\ -\frac{1}{A_{22}} & \frac{C}{A_{22}} \end{bmatrix}$	$h_{12} = -h_{21}$	$\Delta h = 1$
A	$\begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$	$\begin{bmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{bmatrix}$	$\begin{bmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$	$[\boldsymbol{A}]$	$\Delta A = 1$	$A_{11} = A_{22}$

$[\boldsymbol{Z}], [\boldsymbol{Y}], [\boldsymbol{h}], [\boldsymbol{A}]$ **are described in Table 1**.

The simplest two-port network schemes are shown in Figure 2 (a – T two-port network, b – Π two-port network, c – Γ two-port network).



a) b) c)

Figure 2

Example calculation for the admittance matrix of a T two-port network

To calculate the admittance matrix for the two-port network in Figure 2.a) it is simpler to start by calculating the impedance parameters using Table 1 and the equation (2).

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow \begin{matrix} Z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} = Z_1 + Z_2 & Z_{12} = \left.\frac{U_1}{I_2}\right|_{I_1=0} = Z_2 \\ Z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0} = Z_2 & Z_{22} = \left.\frac{U_2}{I_2}\right|_{I_1=0} = Z_2 + Z_3 \end{matrix} \quad (3)$$

Using the conversion from impedance to admittance parameters provided in Table 2 results:

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{Z_2 + Z_3}{(Z_1 + Z_2)(Z_2 + Z_3) - Z_2^2} \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z} = \frac{-Z_2}{(Z_1 + Z_2)(Z_2 + Z_3) - Z_2^2}$$
$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = \frac{-Z_2}{(Z_1 + Z_2)(Z_2 + Z_3) - Z_2^2} \qquad Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{Z_1 + Z_2}{(Z_1 + Z_2)(Z_2 + Z_3) - Z_2^2} \tag{4}$$

With the aid of matrix parameters, the voltage transfer factor from port 1 to port 2 can be determined under an open-circuit condition ($I_2 = 0$):

$$H_{U21g} = \left.\frac{U_2}{U_1}\right|_{I_2=0} = \frac{Z_2}{Z_1 + Z_2} \tag{5}$$

<u>OBSERVATION</u>

To calculate the magnitude and phase of a complex number, $z = x + jy$, the following formulas are applied:

$$|z| = \sqrt{x^2 + y^2}$$

$$\varphi = \arg\{z\} = \arg\{x + jy\} = \begin{cases} \arctan\left(\frac{y}{x}\right), & x > 0 \\ \arctan\left(\frac{y}{x}\right) + \pi, & x < 0 \text{ și } y \geq 0 \\ \arctan\left(\frac{y}{x}\right) - \pi, & x < 0 \text{ și } y < 0 \\ \frac{\pi}{2}, & x = 0 \text{ și } y > 0 \\ -\frac{\pi}{2}, & x = 0 \text{ și } y < 0 \\ indetermination & x = 0 \text{ și } y = 0 \end{cases}$$

For complex numbers ratio:

$$z = \frac{x + jy}{a + jb}$$
$$|z| = \frac{\sqrt{x^2 + y^2}}{\sqrt{a^2 + b^2}}$$
$$\varphi = \arg\{z\} = \arg\{x + jy\} - \arg\{a + jb\}$$

<u>3) Execution of the laboratory tasks</u>

A) **<u>A smontage is made with a purely dissipative Γ two-port network</u>**

On the test board (called "solderless breadboard") the diagram from Figure 3 is designed. Impedances Z_1 and Z_2 are purely dissipative, meaning $Z_1 = R_1$ și $Z_2 = R_2$. (Measure the resistances in the component box with the help of the multimeter and choose them so that $Z_2 > Z_1$).

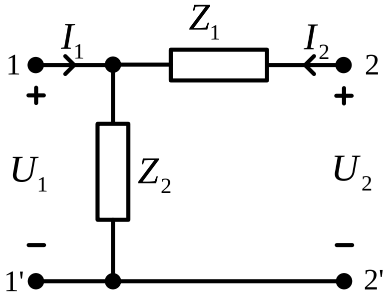


Figure 3

B) **The A-parameters of the attenuator shown in Figure 3 are determined experimentally**

The A-parameters are calculated as ratios involving voltages and currents, quantities measured under certain conditions, as derived from the system equations presented in Table 1 (A-chain parameters). In the current work, only alternating current is used. Voltages are **directly** measured using the multimeter set to the alternating current voltmeter (ACV) function. Current measurements are performed **indirectly** by measuring the voltage drop across a relatively small and known resistance of value.

In order to facilitate indirect current measurement at both port 1 and port 2, resistors are added to the model: R_{a1} at port 1 and R_{a2} at port 2 ($R_{a1} \cong R_{a2} \cong 10\Omega$). The final setup is shown in Figure 4. The R_{a1} and R_{a2} resistance values are measured using the multimeter set to an ohmmeter (attention! R_{a1} and R_{a2} must have negligible effects on the measurements, so they must have smaller values than those of the two-port network. Resistors with values close to 10Ω will be chosen). During impedance measurement, the circuit must not be powered from any signal source!

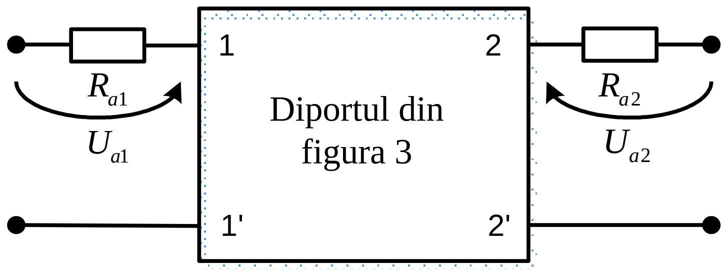


Figure 4

According to the scheme in Figure 4:
- Voltage U_1 is the voltage measured between the terminals 1 și 1';
- Voltage U_2 is the voltage measured between the terminals 2 și 2';
- Current I_1 is determined by dividing the voltage U_{a1} by the resistance value R_{a1}. So $I_1 = \frac{U_{a1}}{R_{a1}}$;
- Current I_2 is determined by dividing the voltage U_{a2} by the resistance value R_{a2}. So $I_2 = \frac{U_{a2}}{R_{a2}}$;

Before starting the measurements, generate a sinusoidal signal with a frequency of 10 kHz and an amplitude of 5 V_{rms}. This signal will remain unchanged until point F.

For measuring the A-parameters proceed as follows:

- **In the case of the condition $I_2 = 0$:**

An open-circuit condition is imposed on port 2. Voltage is applied at port 1 before the additional resistor R_{a1}. Voltage U_1 is measured with the multimeter between the terminals 1 and 1' and voltage U_2 is measured between the terminals 2 and 2'. The voltage across the additional resistor R_{a1} denoted as U_{a1}, is measured with the multimeter and $I_1$is calculated afterwards as $I_1 = \frac{U_{a1}}{R_{a1}}$. All values are recorded in Table 3.

- **In the case of the condition $U_2 = 0$:**

A short-circuit condition is imposed at port 2 after the resistor R_{a2}. Voltage is applied at port 1 before the additional resistor R_{a1}. Voltage U_1 is measured with the multimeter between the terminals 1 and 1'. The voltage across the additional resistor R_{a1} denoted as U_{a1}, is measured with the multimeter and $I_1$is calculated afterwards as $I_1 = \frac{U_{a1}}{R_{a1}}$. The voltage across the additional resistor R_{a2} denoted as U_{a2}, is measured with the multimeter and $I_2$is calculated afterwards as $I_2 = \frac{U_{a2}}{R_{a2}}$. All values are recorded in Table 3.

- **The equation (2) particularized for the A formalism is applied and the elements of the matrix A are determined, ΔA being its determinant.**

Table 3. Experimental determination of A-parameters

Measured values									Calculated values based on measurements				
Condition $I_2 = 0$				Condition $U_2 = 0$					A_{11}	A_{12}	A_{21}	A_{22}	$\boldsymbol{\Delta A}$
U_1	U_2	U_{a1}	I_1	U_1	U_{a1}	I_1	U_{a2}	I_2					
[V]	[V]	*[V]*	[mA]	[V]	[V]	[mA]	*[V]*	[mA]	-	[kΩ]	[mS]	-	-

The error denoted by$\delta = |1 - \Delta \boldsymbol{A}|$ admitted in relation to reciprocity. The admitted error is $\delta \leq 5\%$.

C) **<u>The Z-parameters of the attenuator shown in Figure 3 are determined experimentally</u>**

The method is the same as in point B. All values are recorded in Table 4. The corresponding parameters are ratios involving currents and voltages as indicated by the equations in Table 1 (Z-impedance parameters). They are measured as shown in point B. In the case of the condition I_1=0, an open-circuit condition is imposed on port 1. Voltage is applied at port 2 before the additional resistor R_{a2}.

Table 4. Experimental determination of Z-parameters

Measured values								Calculated values based on measurements			
Condition $I_2 = 0$				Condition $I_1 = 0$				Z_{11}	Z_{12}	Z_{21}	Z_{22}
U_1	U_2	U_{a1}	I_1	U_1	U_2	U_{a2}	I_2				
[V]	[V]	*[V]*	[mA]	[V]	[V]	*[V]*	[mA]	[kΩ]	[kΩ]	[kΩ]	[kΩ]

The error denoted by$\delta = \frac{Z_{12} - Z_{21}}{Z_{12}}$ admitted in relation to reciprocity. The admitted error is $\delta \leq 5\%$.

D) **<u>The Z-parameters are determined by calculation starting from the A-parameters</u>**

Considering the A-parameters determined at point B as accurate, determine the Z'_{ij}-parameters using the conversion equations given in Table 2 (knowing that there is equality between the matrices on the same row). The ' (prime) symbol in Z'_{ij} indicates that the parameters are indirectly determined through calculation from another formalism, distinguishing them from directly measured ones Z_{ij}. Calculate the relative errors between the indirectly determined values and the values directly measured at point C.

$$\delta_{ij} = \frac{|Z'_{ij} - Z_{ij}|}{Z_{ij}}, \qquad \mathrm{i, j} = 1{,}2$$

The admitted error is $\delta_{ij} \leq 5\%$**.**

E) **<u>Theoretical and experimental determination of Y-parameters of the two-port network shown in Figure 5</u>**

For the experimental determination of the Y-parameters, the circuit in Figure 5 (a T two-port network including reactive components) is made on the test board.

Choose the resistor with the value closest to 600 Ω. The capacitance of capacitors C is measured.

To theoretically calculate the Y-parameters, equation (4) is used, replacing the impedances accordingly ($Z_R = R$, $Z_C = \frac{1}{j\omega C}$). For each calculated admittance parameter, its magnitude and phase are theoretically determined.

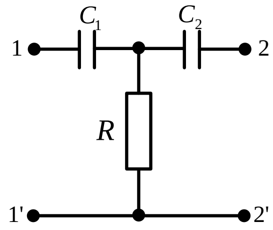


Figure 5

To measure the Y-parameters, proceed as follows :

- **For magnitude measurements**
  - **In the case of the condition $U_2 = 0$:**

A short-circuit condition is imposed at port 2. Voltage is applied at port 1. Voltage magnitude $|U_1|$ is measured with the multimeter between terminals 1 and 1' and current magnitude is calculated afterwards as $|I_1| = \frac{|U_{C_1}|}{|Z_{C_1}(\omega=2\cdot\pi\cdot10000)|}$ across the capacitor C_1. $|Z_{C_1}(\omega = 2\cdot\pi\cdot 10000)|$ is the value of C_1's impedance magnitude at the working frequency (10 kHz). $|I_2|$ it is measured similarly, but through the capacitor C_2. All values are recorded in Table 5.

  - **In the case of the condition $U_1 = 0$:**

A short-circuit condition is imposed at port 1. Voltage is applied at port 2. Voltage magnitude $|U_2|$ is measured with the multimeter between terminals 2 and 2' and current magnitude is calculated afterwards as $|I_2| = \frac{|U_{C_2}|}{|Z_{C_2}(\omega=2\cdot\pi\cdot10000)|}$ across the capacitor C_2. $|Z_{C_2}(\omega = 2\cdot\pi\cdot 10000)|$ is the value of C_2's impedance magnitude at the working frequency (10 kHz). $|I_1|$ it is measured similarly, but through the capacitor C_1. All values are recorded in Table 5.

Table 5 Experimental determination of the magnitudes of the Y-parameters

Measured values										Calculated values based on measurements			
Condition $U_2 = 0$					Condition $U_1 = 0$					$\|Y_{11}\|$	$\|Y_{12}\|$	$\|Y_{21}\|$	$\|Y_{22}\|$
$\|U_1\|$	$\|U_{C1}\|$	$\|I_1\|$	$\|U_{C2}\|$	$\|I_2\|$	$\|U_2\|$	$\|U_{C1}\|$	$\|I_1\|$	$\|U_{C2}\|$	$\|I_2\|$				
[V]	*[V]*	[mA]	*[V]*	[mA]	[V]	*[V]*	[mA]	*[V]*	[mA]	[mS]	[mS]	[mS]	[mS]

- **For phase measurements**
  - **In the case of the condition $U_2 = 0$:**

A short-circuit condition is imposed at port 2. Voltage is applied at port 1. The argument $\arg\{Y_{11}\}$ is the phase shift between the current I_1 and voltage $U_1$when $U_2 = 0$. Current I_1 cannot be measured directly. Instead, the phase shift between the voltage across capacitor C_1 and voltage U_1 will be measured using an oscilloscope using the method of synchronization with a reference signal. Given that the phase shift between the voltage across capacitor C_1 and the current through it is equal to −90°, the sought phase difference can be determined.

The phase shift between the voltage across the capacitor C_1 (U_{C_1}) and the voltage U_1 is measured as follows:

- Connect channel 1 of the oscilloscope to measure the voltage U_1. Press AUTOSET;
- Activate averaging acquisition mode (for noise reduction): press the ACQUIRE button and select Average mode, then choose 16 for Averages.
- Adjust channel 1 parameters (press CH1 button) as follows: Coupling: DC, BW Limit: Full, Volts/Div: Coarse, Probe: 1X, Invert: Off. Adjust the vertical deflection coefficient for CH1 to 2V/div from the VOLTS/DIV knob (CH1 2.00V on the right side of the oscilloscope screen);
- Adjust synchronization parameters as follows: Trigger Setup → Type: Edge, Source: CH1, Slope: Rising, Mode: Auto, Coupling: DC. Adjust the Trigger level to 0 (use the LEVEL rotary knob until 0.00V is displayed in the bottom-right part of the oscilloscope screen);
- Connect channel 2 of the oscilloscope between the two capacitors so that the difference CH1−CH2 represents the voltage on the capacitor $C_1(U_{C_1})$;
- Adjust channel 2 parameters (press the CH2 button) as follows: Coupling: DC, BW Limit: Full, Volts/Div: Coarse, Probe: 1X, Invert: Off. Adjust the vertical deflection coefficient for CH2 to 2V/div from the VOLTS/DIV knob. (CH2 2.00V on the right side of the oscilloscope screen);
- Press the MATH button of the oscilloscope and select: Operator: −, SourceA: CH1, SourceB: CH2, Scale: 2V/div;
- Adjust the HORIZONTAL POSITION (from the rotary knob) until Delay: 0.00s is displayed in the top-right part of the oscilloscope screen;
- Activate the cursors by pressing the CURSORS button, Source: MATH and the option for measuring on Ox axis (button X). With the Intensity/Adjust rotary knob position one cursor at the first zero-crossing of the signal, from bottom to top, located in the middle-right of the screen. Read the value indicated next to the respective cursor and note it as $\Delta t_{U_{C1}-U_1}$ (see Figure 6) and record it in Table 6.
- Calculate the phase shift between the voltage U_{C_1} and U_1 as: $\varphi_{U_{C1}-U_1} = -360° \cdot \Delta t_{U_{C1}-U_1} \cdot f$;
- Calculate the phase shift between the current through capacitor $I_{C_1} = I_1$ and input voltage U_1 using the equation: $\varphi_{I_1-U_1} = \varphi_{U_{C1}-U_1} + 90° = arg\{Y_{11}\}$, the sought quantity. Record it in Table 6.

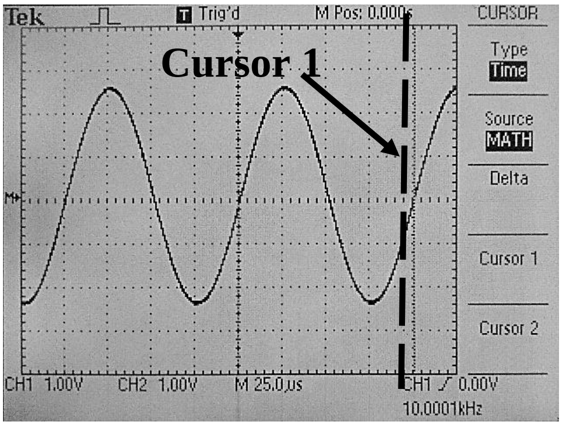


Figure 6

The argument $\arg\{Y_{21}\}$ is the phase shift between the current I_2 and voltage $U_1$when $U_2 = 0$. Current I_2 cannot be measured directly. Instead, the phase shift between the voltage across capacitor C_2 and voltage U_1 will be measured using an oscilloscope using the method of synchronization with a reference signal. **The connections and adjustments made in the previous step are assumed to be completed.**

The phase shift between the voltage across the capacitor C_2 (U_{C_2}) and the voltage U_1 is measured as follows:

- Adjust channel 2 Invert: On parameter (press the CH2 button). Since $U_2 = 0$, the waveform displayed on channel 2 under these conditions represents U_{C2} taken in t appropriate direction (that of current I_2);
- Press the MATH button repeatedly until the respective waveform disappears;

- Activate the cursors by pressing the CURSORS button, Source: CH2 and the option for measuring on Ox axis (button X). With the Intensity/Adjust rotary knob position one cursor at the first zero-crossing of the signal, from bottom to top, located in the middle-right of the screen. Read the value indicated next to the respective cursor and note it as $\Delta t_{U_{C2}-U_1}$ (see Figure 7) and record it in Table 6.
- Calculate the phase shift between the voltage U_{C_2} and U_1 as: $\varphi_{U_{C2}-U_1} = -360° \cdot \Delta t_{U_{C2}-U_1} \cdot f$;
- Calculate the phase shift between the current through capacitor $I_{C_2} = I_2$ and input voltage U_1 using the equation: $\varphi_{I_2-U_1} = \varphi_{U_{C2}-U_1} + 90° = arg\{Y_{21}\}$, the sought quantity. Record it in Table 6.

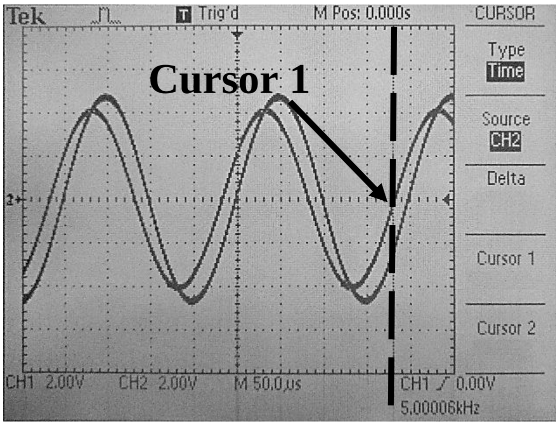


Figure 7

- **In the case of the condition $U_1 = 0$**

A short-circuit condition is imposed at port 1. Voltage is applied at port 2. The argument arg$\{Y_{22}\}$ is the phase shift between the current I_2 and the voltage U_2 when $U_1 = 0$ Current I_2 cannot be measured directly. Instead, the phase shift between the voltage across capacitor C_2 and voltage U_1 will be measured using an oscilloscope using the method of synchronization with a reference signal. Given that the phase shift between the voltage across capacitor C_1 and the current through it is equal to −90°, the sought phase difference can be determined.

The procedure is the same as in the case of condition $U_2 = 0$, except that all indices 1 become 2, and vice versa. The properties of the measured two-port will be taken into account to verify the results obtained after each measurement.

Table 6 Experimental determination of the phases of the Y-parameters

Measured values				Calculated values based on measurements			
Condition $U_2 = 0$		Condition $U_1 = 0$		$arg\{Y_{11}\}$	$arg\{Y_{21}\}$	$arg\{Y_{22}\}$	$arg\{Y_{12}\}$
$\Delta t_{U_{C1}-U_1}$	$\Delta t_{U_{C2}-U_1}$	$\Delta t_{U_{C2}-U_2}$	$\Delta t_{U_{C1}-U_2}$				
[μs]	[μs]	[μs]	[μs]	[°]	[°]	[°]	[°]

Calculate the relative errors between the theoretical and measured values.

F) <u>Experimental determination of the magnitude of the voltage transfer factor</u>

For the two-port network shown in Figure 5, determine the magnitude of the theoretical voltage transfer factor from port 1 to port 2, when the latter is evaluated under open-circuit condition, at various frequencies. Compare the theoretical and experimental values by completing Table 7. Plot both sets of data on the same graph. What type of filter is obtained?

- A sinusoidal voltage is applied to port 1. The voltages at the both ports are measured using the multimeter set as an alternating current voltmeter (ACV) at various frequencies. Their ratio represents the magnitude of the voltage transfer factor at each frequency.

Tabel 7

f [kHz]	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5
U_2 $[V]$										
$\lvert H_{U21g}\rvert = \frac{U_2}{U_1}$										
$\lvert H_{U21g}\rvert_t$										

G) **Experimental determination of the phases of the voltage transfer factor**

For the two-port network shown in Figure 5, determine the phase of the theoretical voltage transfer factor from port 1 to port 2, when the latter is evaluated under open-circuit condition, at various frequencies:

- Connect channel 1 of the oscilloscope to measure the voltage U_1. Connect channel 2 of the oscilloscope to measure the voltage U_2. Press AUTOSET;
- Adjust the same vertical deflection coefficients for both channels from the VOLTS/DIV knobs;
- Adjust synchronization parameters as follows: Trigger Setup → Type: Edge, Source: CH1, Slope: Rising, Mode: Auto, Coupling: DC. Adjust the Trigger level to 0 (use the LEVEL rotary knob until 0.00V is displayed in the bottom-right part of the oscilloscope screen);
- Adjust the HORIZONTAL POSITION (from the rotary knob) until Delay: 0.00s is displayed in the top part of the oscilloscope screen;
- Activate the cursors by pressing the CURSORS button, Source: CH2 and the option for measuring on Ox axis (button X). With the Intensity/Adjust rotary knob position one cursor at the first zero-crossing of the signal, from bottom to top of the signal connected to channel 2, located in the middle-right of the screen. Read the value indicated next to the respective cursor and note it as Δt and record it in Table 8.
- Calculate the phase shift as: $\varphi = -360° \cdot \Delta t \ \cdot f$;

Table 8

f [kHz]	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5
Δt										
φ [degrees]										
$\varphi_{teoretic}$ [degrees]										

Compare the theoretical and experimental values by completing Table 8. Plot both sets of data on the same graph.

H), I), J) **Points E), F), G) are repeated for the two-port network shown in Figure 8.** R_1 and R_2 are chosen to be equal, equal to the resistance with the value closest to 600 Ω.

Pay attention when measuring the phase shifts of the Y-parameters!

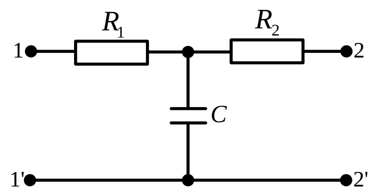


Figure 8